6.3 Complex Fractions

A. Introduction

Fractions that have fractions in the numerator and/or denominator are **complex fractions**.

For example, $\dfrac{x-\frac{x}{y}}{4+\frac{1}{x^2}}$ is a complex fraction.

To simplify these, we treat them as three problems in one.

“Do the numerator problem; do the denominator problem; then do the division.”

B. Examples

Example 1: Simplify $\dfrac{x-\frac{x}{y}}{4+\frac{1}{x^2}}$

Solution

1. Numerator problem:

$$x-\frac{x}{y}=\frac{x}{1}-\frac{x}{y}=\frac{xy}{y}-\frac{x}{y}=\frac{xy-x}{y}=\frac{x(y-1)}{y}$$

2. Denominator problem:

$$4+\frac{1}{x^2}=\frac{4}{1}+\frac{1}{x^2}=\frac{4x^2}{x^2}+\frac{1}{x^2}=\frac{4x^2+1}{x^2}$$

3. Division step:

$$\frac{x-\frac{x}{y}}{4+\frac{1}{x^2}}=\frac{\frac{x(y-1)}{y}}{\frac{4x^2+1}{x^2}}=\frac{x(y-1)}{y}\div\frac{4x^2+1}{x^2}$$

$$\frac{x(y-1)}{y} \cdot \frac{x^2}{4x^2+1}$$

Ans $\boxed{\dfrac{x^3(y-1)}{y(4x^2+1)}}$

Example 2: Find $\dfrac{\frac{4x}{x-3}+\frac{2}{3-x}}{\frac{5}{x}+\frac{3}{x-3}}$

Solution

1. Numerator Problem:

$$\frac{4x}{x-3}+\frac{2}{3-x}$$

$$\frac{4x}{x-3}-\frac{2}{x-3} \qquad \text{(negative factor pairs)}$$

$$\frac{4x-2}{x-3}$$

$$\frac{2(2x-1)}{x-3}$$

2. Denominator Problem:

$$\frac{5}{x}+\frac{3}{x-3}=\frac{5(x-3)}{x(x-3)}+\frac{3x}{x(x-3)}=\frac{5x-15}{x(x-3)}+\frac{3x}{x(x-3)}=\frac{8x-15}{x(x-3)}$$

3. Division step:

$$\frac{\frac{4x}{x-3}+\frac{2}{3-x}}{\frac{5}{x}+\frac{3}{x-3}}=\frac{\frac{2(2x-1)}{x-3}}{\frac{8x-15}{x(x-3)}}=\frac{2(2x-1)}{x-3}\div\frac{8x-15}{x(x-3)}$$

$$\frac{2(2x-1)}{x-3} \cdot \frac{x(x-3)}{8x-15}$$

$$\frac{2(2x-1)}{\cancel{x-3}} \cdot \frac{x\cancel{(x-3)}}{8x-15}$$

Ans $\boxed{\frac{2x(2x-1)}{8x-15}}$