3.5 Relations and Functions: Basics

A. Relations

1. A **relation** is a set of ordered pairs. For example,

   $A = \{(-1,3),\ (2,0),\ (2,5),\ (-3,2)\}$

2. **Domain** is the set of all first coordinates: $\{-1,2,2,-3\}$

   so $\mathit{dom}(A) = \{-1,2,-3\}$

3. **Range** is the set of all second coordinates: $\{3,0,5,2\}$

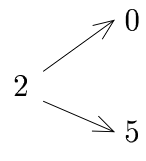
   so $\mathit{rng}(A) = \{3,0,5,2\}$

B. Functions

A **function** is a relation that satisfies the following:

each x-value is allowed only **one** y-value

Note: A (above) is **not** a function, because 2 has y-values 0 and 5 **(violates our condition!)**



bad!

C. Testing Relations To See If They Are Functions

We make a "mapping table". We do this as follows:

1. List all the x-values on the left.

2. At each x-value, draw an arrow–one arrow pointing to each y-value it has.

3. If you see a situation where an x-value has two or more arrows branching to y-values, then it is **not** a function.

Examples:

Check to see if the following relations are functions:

$$B = \{(3,4),\ (2,4),\ (1,4),\ (-3,2)\}$$
$$C = \{(1,2),\ (-2,3),\ (5,1),\ (1,4)\}$$

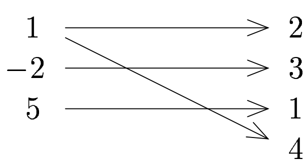
Solution

Make a mapping table for B:

$$\begin{array}{rcl} 3 & \longrightarrow & 4 \\ 2 & \longrightarrow & 4 \\ 1 & \longrightarrow & 4 \\ -3 & \longrightarrow & 2 \end{array}$$

Thus we see that B is a function.

Make a mapping table for C:



Thus we see that C is **not** a function!

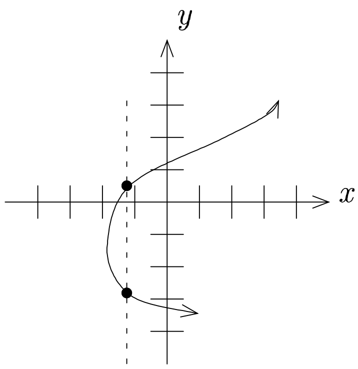
C. Graphs and Functions

To check to see if a **graph** determines a function, we apply the **Vertical Line Test**.

Vertical Line Test:

> If a vertical line moved over allowed x-values intersects the graph exactly once (each time), the graph is a function; otherwise; it is not.

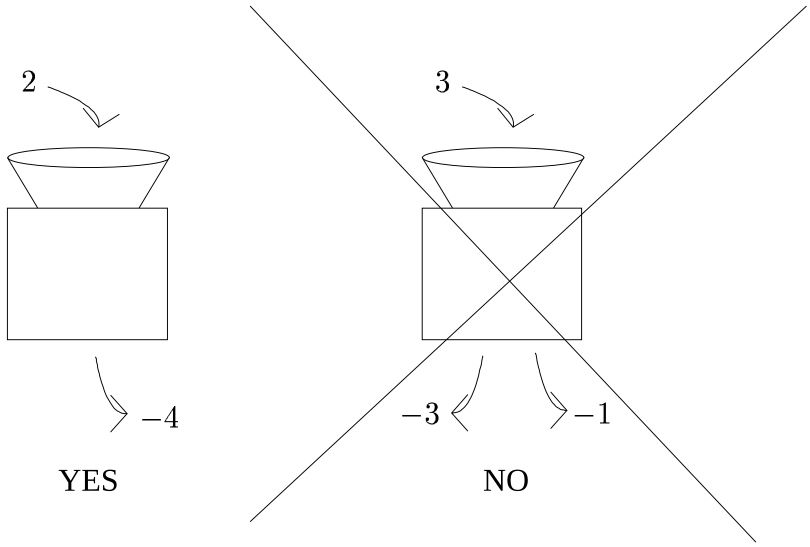
Example:



not a function!

D. “Function Machine”

Since each x-value is allowed only **one** y-value (in a function), we can think of a function as a machine that “eats” x-values and spits back y-values–so that the machine only spits out one output for any input.



E. Function Notation

We call our “machine” that changes x-values into y-values a function operator, written f.

In other words, f represents the function

Thus, since $B = \{(3,4),\ (2,4),\ (1,4),\ (-3,2)\}$ is a function, we can write

$$
\begin{aligned}
f(3) &= 4\\
f(2) &= 4\\
f(1) &= 4\\
f(-3) &= 2
\end{aligned}
$$

F. Comments on Function Notation

1. $\mathscr{f}$ here is **not** multiplication; it is function operation.

2. To avoid confusion with variables, we write functions in **cursive**.

   Thus, we write $\mathscr{f}$, $\mathscr{g}$, $\mathscr{h}$ rather than f, g, h.

3. In general:

$$y = \mathscr{f}(x)$$

y: the output

x: the input

$\mathscr{f}$: the function operator

4. **Note:** $\mathscr{f}$ is the function operator; but $\mathscr{f}(x)$ is the **output** (same as y!)

G. Function Evaluation

Sometimes a function has an output formula given by $f(x)$.

To evaluate the output for f, given an input:

We just plug in the input, wherever we see x.

Example 1: Given $f(x) = 6 - x^2$. Find $f(1)$ and $f(-2)$.

Solution

$f(1)$: plug in 1 where you see x:

$$f(1) = 6 - (1)^2 = 6 - 1 = \boxed{5}$$

$f(-2)$: plug in -2 where you see x:

$$f(-2) = 6 - (-2)^2 = 6 - 4 = \boxed{2}$$

Example 2: Given $f(x) = 2x^2 - 4x + 6$. Find $f(0)$ and $f\left(-\frac{1}{2}\right)$.

Solution

$f(0)$: plug in 0 where you see x:

$$f(0) = 2(0)^2 - 4(0) + 6 = 2 \cdot 0 - 4 \cdot 0 + 6 = 0 - 0 + 6 = \boxed{6}$$

$f\left(-\frac{1}{2}\right)$: plug in $-\frac{1}{2}$ where you see x:

$$
\begin{aligned}
f\left(-\frac{1}{2}\right) &= 2\left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) + 6 \\
&= 2 \cdot \frac{1}{4} + 2 + 6 \\
&= \frac{1}{2} + 2 + 6 \\
&= \frac{1}{2} + \frac{4}{2} + \frac{12}{2} \\
&= \boxed{\frac{17}{2}}
\end{aligned}
$$

Example 3: Given $f(x) = \sqrt{x-3}$. Find $f(3a+b)$.

Solution

$f(3a+b)$: plug in $(3a+b)$ where you see x:

$$f(3a+b) = \boxed{\sqrt{(3a+b)-3}}$$