1.1 Real Numbers

A. Sets

A **set** is a list of numbers: $\{3, 7, 15, \frac{2}{3}, \pi, -10\}$

We separate the entries with commas, and close off the left and right with $\{$ and $\}$.

The **empty set** is the set containing nothing: $\{\}$. It is given the symbol $\emptyset$.

B. Special Sets

1. **Natural Numbers**: $\mathbb{N} = \{1, 2, \ldots\}$ (these are the counting numbers)

2. **Whole Numbers**: $\mathbb{W} = \{0, 1, 2, \ldots\}$ (same as $\mathbb{N}$, but throw in zero)

3. **Integers**: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

   (same as the whole numbers, but allow negative whole numbers)

4. **Rational Numbers**: $\mathbb{Q}$

   This contains:

   a. all integers, like -2, 5, or 0

   b. all fractions positive and negative, like $\frac{3}{4}$ and $-\frac{7}{5}$

   c. all decimals that terminate (stop), like $.25$

   d. all decimals that have a repeating block, like $.1845454545\ldots$

   It is a fact that all rational numbers can be turned into fractions: stay tuned

5. **Irrational Numbers**: $\mathbb{R} \backslash \mathbb{Q}$

This contains only infinite decimals that don't have a repeating block.

Examples:

$\pi = 3.141592\ldots$

$-\sqrt{3} = -1.73205\ldots$

$-1.1010010001\ldots$

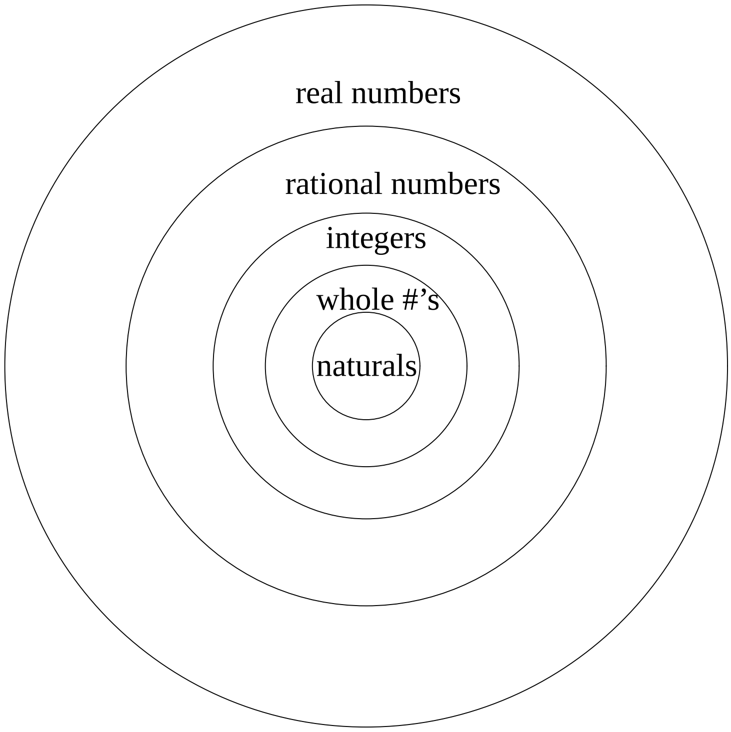
It is a fact that all roots that don't come out "cleanly" are irrational.

Thus $\sqrt{5}$, $\sqrt[3]{7}$, $-\sqrt[5]{11}$ are all irrational.

6. **Real Numbers**: $\mathbb{R}$

This consists of anything that is either rational or irrational. Hence all numbers (in this course) are real.

C. Picture of Special Sets



Notice that irrational numbers are in a separate group.

D. Practice Problems on Special Sets

Try to decide which sets each number belongs to. A number may belong to only one set, or several; list all possible.

1. 6

2. $-\frac{3}{4}$

3. $\sqrt{6}$

4. 0

5. $.89$

6. $-.89121212\ldots$

7. $.123456789101112\ldots$

8. $\sqrt{9}$

E. Properties of Real Numbers

1. **Commutative**: we can add or multiply in any **order**

   $3 + 4 = 4 + 3$ commutative property of addition

   $3 \cdot 4 = 4 \cdot 3$ commutative property of multiplication (note: $\cdot$ means multiply)

2. **Associative**: In repeated adding or multiplying, we can **move parentheses**

   $(2 + 3) + 4 = 2 + (3 + 4)$ associative property of addition

   $(5 \cdot 6) \cdot 7 = 5 \cdot (6 \cdot 7)$ associative property of multiplication

   Parenthesis mean **do this first**

3. **Identity**: Adding zero or multiplying by 1 **does nothing**

   $2 + 0 = 2$ identity property of addition

   $5 \cdot 1 = 5$ identity property of multiplication

   Note: 0 is called the **additive identity**, and 1 is called the **multiplicative identity**.

4. **Inverse**: Two numbers **add to zero** or **multiply to 1**

   $6 + (-6) = 0$ inverse property of addition

   $6 \cdot \frac{1}{6}$ inverse property of multiplication

   Note: -6 is the **additive inverse**; $\frac{1}{6}$ is the **multiplicative inverse** (reciprocal)

5. **Distributive**: Multiplying a sum

   $3(4 + 6) = 3 \cdot 4 + 3 \cdot 6$ distributive property of multiplication over addition

F. Practice Problems on Properties

For each mathematical statement below, correct identify the name of the property shown.

1. $8 + 0 = 8$

2. $(5 \cdot 2) \cdot \sqrt{7} = 5 \cdot (2 \cdot \sqrt{7})$

3. $4(\sqrt{3} + \frac{1}{3}) = 4 \cdot \sqrt{3} + 4 \cdot \frac{1}{3}$

4. $3 \cdot \frac{1}{3} = 1$

5. $2.4 + 5 = 5 + 2.4$

6. $7 \cdot 1 = 7$

8. $4 + (-4) = 0$

9. $\frac{2}{5}(6 + \sqrt{2}) = \frac{2}{5} \cdot 6 + \frac{2}{5} \cdot \sqrt{2}$