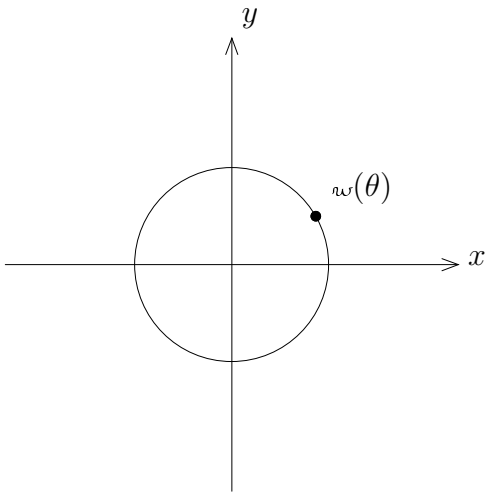
T3.8 Trigonometric Functions: Periodicity

A. Introduction



Since ω spits back the same point every time we add 2π, we say that ω is **periodic**.

B. Periodicity

Formally, a function f is said to be **periodic** if $f(x + p) = f(x)$ for some p. The smallest such value of p that makes the function periodic is called the **period**.

C. Periodicity of the Wrapping Function

By the above discussion, $\omega(\theta + 2\pi) = \omega(\theta)$, so the wrapping function is periodic. From Section 3.1C, we see that 2π is the smallest such value, so ω has period 2π.

D. Periodicity of the Trigonometric Functions

Since the trigonometric functions are defined in terms of ω, they are also periodic, and repeat every 2π.

Note: If $\omega(\theta) = (x, y)$, then $\omega(\theta + \pi) = (-x, -y)$ so, in particular, tangent and cotangent actually repeat every π.

E. Summary of Periodicity

	Period
$\sin$	2π
$\cos$	2π
$\tan$	π
$\cot$	π
$\sec$	2π
$\csc$	2π