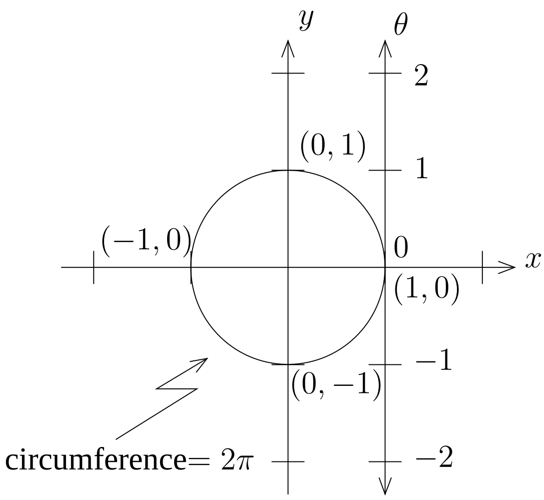
T3.2 The Wrapping Function

A. Setup

Put a number line in vertical position at $(1, 0)$ on the unit circle.

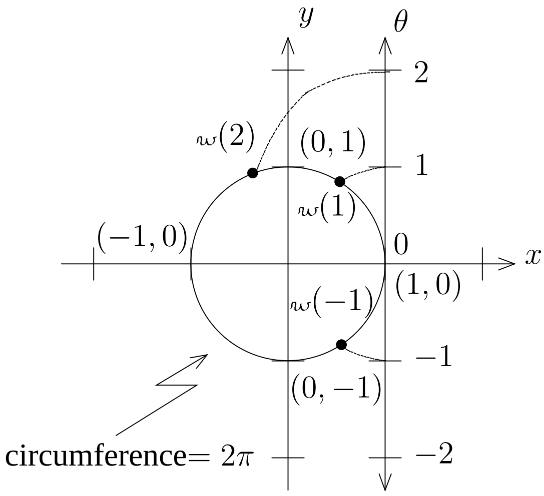


Let θ (theta) be the number line variable.

Now “wrap” the number line around the circle.

ω: wrapping function

$\omega(\theta)$: point on the unit circle where θ on the number line wraps to



Note: $\omega(\theta)$ is an ordered pair in the plane and not a number.

B. Strategy

To easily and semiaccurately locate $\omega(\theta)$, we use the following guidelines:

1. Every π wraps halfway around the circle counterclockwise and every $-\pi$ wraps halfway around the circle clockwise.

2. To locate $\omega(\theta)$ when θ is a fractional multiple of π, we divide the semicircle into fractional parts. For instance, if (in lowest terms), we want to find $\omega(\frac{m\pi}{n})$, then we divide the semicircle into n equal sized wedges and count to the correct wedge, namely the mth one.

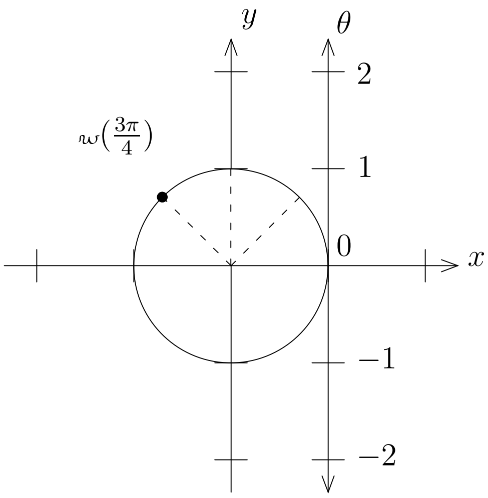
3. To locate $\omega(\theta)$ when θ is an integer, we use the number line as a guide and recognize that $\pi \approx 3.14$. Thus $\omega(3)$ is slightly above $(-1,0)$ on the unit circle.

C. Examples

Example 1: Locate and mark $w(\frac{3\pi}{4})$ on the unit circle.

Solution

We divide up the upper semicircle into four equal wedges and count over to the third wedge:

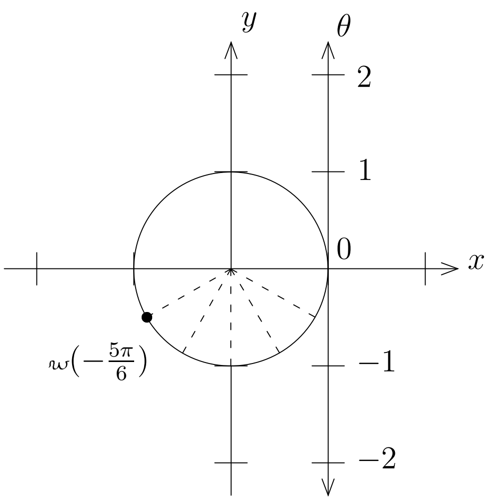


Example 2: Locate and mark $\omega(-\frac{5\pi}{6})$ on the unit circle.

Solution

Here we are wrapping clockwise, starting on the bottom side of the unit circle.

We divide up the lower semicircle into six equal wedges and count clockwise to the fifth wedge:



Example 3: Locate and mark $\wp(\frac{11\pi}{3})$ on the unit circle.

Solution

Here we are wrapping counterclockwise, starting on the top side of the unit circle. Note that $\frac{11\pi}{3} = 3\frac{2}{3}\,\pi$. Now 2π takes us once around, so 3π takes us halfway around to $(-1, 0)$. Then we need to go $\frac{2}{3}\pi$ more. Hence, we divide up the lower semicircle into three equal wedges and move to the second wedge.