T3.1 Circles and Revolutions

A. Circles

Standard Form: $(x-a)^2 + (y-b)^2 = r^2$

1. center: (a, b)

2. radius: r

3. circumference: $2\pi r$

B. Examples

Example 1: Find the center, radius, circumference, x and y intercepts of the circle, where $(x-1)^2+(y+4)^2=12$. Then sketch the circle.

Solution

center: $\boxed{(1,-4)}$

radius: $\sqrt{12}=\boxed{2\sqrt{3}}$

circumference: $2\pi\cdot 2\sqrt{3}=\boxed{4\pi\sqrt{3}}$

x**-intercepts:** set $y=0$:

$$\begin{aligned}(x-1)^2+(0+4)^2&=12\\(x-1)^2+16&=12\\(x-1)^2&=-4\\x-1&=\pm\sqrt{-4}\\x-1&=\pm 2i\\x&=1\pm 2i\end{aligned}$$

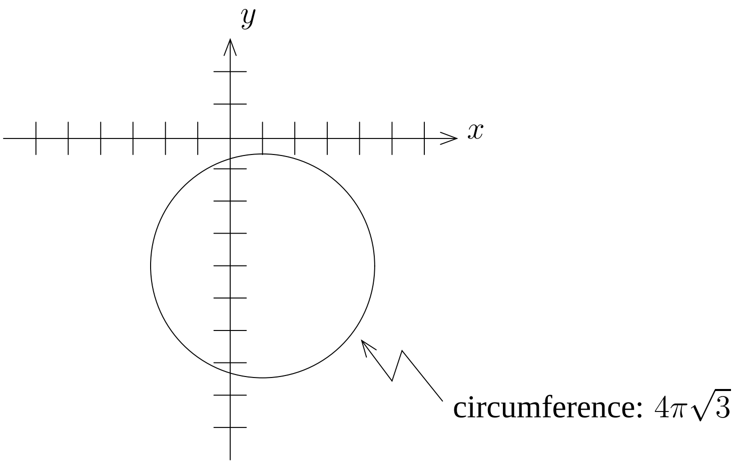
$\boxed{\text{Thus there are no } x\text{-intercepts.}}$

y**-intercepts:** set $x=0$:

$$\begin{aligned}(0-1)^2+(y+4)^2&=12\\1+(y+4)^2&=12\\(y+4)^2&=11\\y+4&=\pm\sqrt{11}\\y&=-4\pm\sqrt{11}\end{aligned}$$

$\boxed{\text{Thus the } y\text{-intercepts are } -4\pm\sqrt{11}.}$

Graph:



Example 2: Find the center, radius, circumference, x and y intercepts of the circle, where $x^2 + y^2 = 1$. Then sketch the circle.

Solution

center: $\boxed{(0,0)}$

radius: $\sqrt{1} = \boxed{1}$

circumference: $2\pi \cdot 1 = \boxed{2\pi}$

x**-intercepts:** set $y = 0$:

$$\begin{aligned} x^2 + 0^2 &= 1 \\ x^2 &= 1 \\ x &= \pm 1 \end{aligned}$$

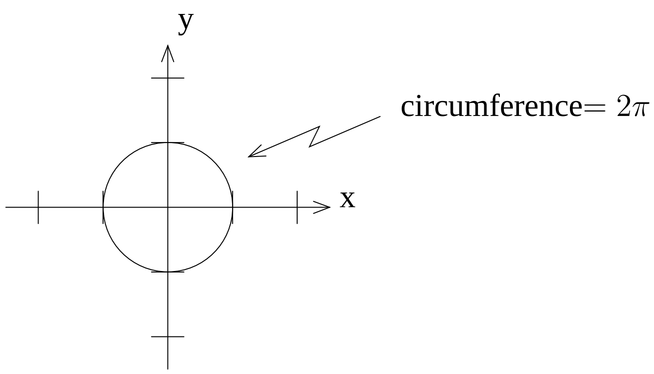
$\boxed{\text{Thus the } x\text{-intercepts are } \pm 1.}$

y**-intercepts:** set $x = 0$:

$$\begin{aligned} 0^2 + y^2 &= 1 \\ y^2 &= 1 \\ y &= \pm 1 \end{aligned}$$

$\boxed{\text{Thus the } y\text{-intercepts are } \pm 1.}$

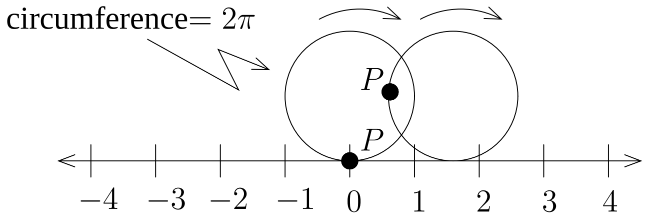
Graph:



Note: This special circle is called the **unit circle**.

C. Revolutions on the Number Line

We put a unit circle on the number line at 0, and allow the circle to “roll” along the number line:



Note: The circle will touch 2π (at the point P) on the number line when the circle rolls to the right one complete revolution.

D. Examples

Where does the circle touch the number line if the circle rolls . . .

Example 1: To the right, two complete revolutions?

Solution

Ans $\boxed{4\pi}$

Example 2: To the left, one complete revolution?

Solution

Ans $\boxed{-2\pi}$

Example 3: To the right, half a complete revolution?

Solution

Ans $\boxed{\pi}$

Example 4: To the left, one and a half complete revolutions?

Solution

Ans $\boxed{-3\pi}$
