

4.3 ~~E~~ Change of Base Formula

Suppose we wish to convert from one base to another in a logarithm

A. Formula

$$\log_{\text{old}} N = \frac{\log_{\text{new}} N}{\log_{\text{new}} \text{old}}$$

B. Derivation

This derivation is worth knowing!

Given $\log_o N$:

$$\text{Let } x = \log_o N$$

Apply $o^{\square}$ to both sides: $o^x = \cancel{o^{\log_o N}}$

$$o^x = N$$

Now apply $\log_n$ to both sides:

$$\log_n o^x = \log_n N$$

$$x \log_n o = \log_n N$$

power rule

$$x = \frac{\log_n N}{\log_n o}$$

$$\text{Thus } \log_o N = \frac{\log_n N}{\log_n o}.$$

C. Natural Logs

Using e as the new base:

$$\log_b N = \frac{\ln N}{\ln b}$$

D. Examples

Example 1: Change $\log_3 7$ to base 8

Solution

$$\log_{\text{old}} N = \frac{\log_{\text{new}} N}{\log_{\text{new old}}}$$

$$\Rightarrow \log_3 7 = \frac{\log_8 7}{\log_8 3}$$

Ans: $\boxed{\frac{\log_8 7}{\log_8 3}}$

Example 2: Change $\log_6 15$ to natural logs

Solution

$$\log_b N = \frac{\ln N}{\ln b}$$

$$\Rightarrow \log_6 15 = \frac{\ln 15}{\ln 6}$$

Ans: $\boxed{\frac{\ln 15}{\ln 6}}$

E. Comments

1. We can use the change of base formula to convert logarithms to natural logs. Thus we can evaluate any logarithm on a calculator using the "ln" button.

2. Example: Evaluate $\log_9 4$ on a calculator

Solution

Change to natural logs: $\log_9 4 = \frac{\ln 4}{\ln 9}$

⚡
Now use the calculator

$$\approx \boxed{.6309}$$