

4.3C Using Logarithm Properties: Expanding

A. Summary of Logarithm Properties

1. Product Rule: $\log_b M + \log_b N = \log_b(MN)$

2. Quotient Rule: $\log_b M - \log_b N = \log_b\left(\frac{M}{N}\right)$

3. Power Rule: $p \log_b M = \log_b(M^p)$

B. Expanding Logarithms

Here we go backward.

Given one single complicated logarithm, we expand into several smaller logarithms.

This can be more difficult for two reasons:

1. We need to use the logarithm properties backward.

2. It can be difficult to see what rule to use.

C. Determining the Right Rule to Use in Expansions

Imagine plugging numbers into the expression and evaluating step-by-step. The last operation you do before applying the logarithm, gives you the name of the rule to use.

D. Method for Expanding Logarithms

1. Convert all roots/radicals to fractional powers.
2. Determine the correct rule to use, as in Part C.
3. Apply the rule.
4. To each new logarithm you see, repeat the procedure.
5. An individual logarithm can not be expanded further if there is no legitimate property to use.

E. Examples

Example: Expand the logarithm: $\ln\left(\sqrt[3]{\frac{1-x}{1+x}}\right)$

Solution

1. Convert radical: $\ln\left[\left(\frac{1-x}{1+x}\right)^{\frac{1}{3}}\right]$

2. Last operation: Power

3. Use Power Rule (backward):

$$\frac{1}{3} \ln\left(\frac{1-x}{1+x}\right)$$

4. Now repeating; last operation: Quotient

$$\frac{1}{3} [\ln(1-x) - \ln(1+x)]$$

$$\boxed{\frac{1}{3} \ln(1-x) - \ln(1+x)} \quad \underline{\text{Ans}}$$

can't do anything!

Example 2: Expand the logarithm: $\lg(x^2 \sqrt[3]{x+4})$

Solution

$$\lg(x^2 \sqrt[3]{x+4}) = \lg(x^2 (x+4)^{1/3})$$

$$= \lg(x^2) + \lg((x+4)^{1/3}) \quad \text{Product Rule}$$

$$= 2\lg x + \frac{1}{3}\lg(x+4) \quad \text{Power Rule}$$

↑
can't expand: false property!

$$\text{Ans: } \boxed{2\lg x + \frac{1}{3}\lg(x+4)}$$

Example 3: Expand the logarithm: $\log_{1/2}\left(\left(\frac{1}{2}\right)^x x^{1/2}\right)$

Solution

$$\log_{1/2}\left(\left(\frac{1}{2}\right)^x x^{1/2}\right) = \log_{1/2}\left(\left(\frac{1}{2}\right)^x\right) + \log_{1/2}(x^{1/2}) \quad \text{Product Rule}$$

$$= x + \log_{1/2}(x^{1/2}) \quad \text{Annihilating Property}$$

$$= x + \frac{1}{2}\log_{1/2} x \quad \text{Power Rule}$$

$$\text{Ans: } \boxed{x + \frac{1}{2}\log_{1/2} x}$$