4.3 B Using Logarithm Properties: Condensing

A. Summary of Logarithm Properties

1. Product Rule: $\log_b M + \log_b N = \log_b(MN)$

2. Quotient Rule: $\log_b M - \log_b N = \log_b\left(\frac{M}{N}\right)$

3. Power Rule: $p\log_b M = \log_b(M^p)$

B. Condensing Logarithms

Given a combination of several logarithms, we condense into one single logarithm.

Method: 1. Use Power Rule first to make all logarithm coefficients equal to 1.
2. Then use Product/Quotient Rules reading left to right

C. Examples

Example1: Condense the logarithms: $\frac{1}{3}\log_7(x+2) - \frac{1}{8}\log_7(2x-1)$

Solution

$$\frac{1}{3}\log_7(x+2) - \frac{1}{8}\log_7(2x-1)$$

$$= \log_7\left[(x+2)^{\frac{1}{3}}\right] - \log_7\left[(2x-1)^{\frac{1}{8}}\right] \quad \text{Power Rule}$$

$$= \boxed{\log_7\left(\frac{(x+2)^{\frac{1}{3}}}{(2x-1)^{\frac{1}{8}}}\right)} \quad \text{Ans}$$

Example 2: Condense the logarithms: $3\lg x - 4\lg 2 + \lg 5$

Solution

$$3\lg x - 4\lg 2 + \lg 5$$

$$= \lg(x^3) - \lg(2^4) + \lg 5 \quad \text{Power Rule}$$

$$= [\lg(x^3) - \lg(16)] + \lg 5$$

$$= \lg\left(\frac{x^3}{16}\right) + \lg 5 \quad \text{Product Rule}$$

$$= \lg\left(\frac{x^3}{16} \cdot 5\right)$$

Ans: $\boxed{\lg\left(\frac{5x^3}{16}\right)}$

Example 3: Condense the logarithms:

$$\frac{2}{3}\left(\ln(x^3) - \ln y\right) - \frac{1}{3}\ln(z+1)$$

Solution

Can't use the power rule on the $\frac{2}{3}$ yet because it is not multiplying a single logarithm!

Distribute:

$$\frac{2}{3}\ln(x^3) - \frac{2}{3}\ln y - \frac{1}{3}\ln(z+1)$$

$$= \ln\left[(x^3)^{\frac{2}{3}}\right] - \ln\left(y^{\frac{2}{3}}\right) - \ln\left[(z+1)^{\frac{1}{3}}\right] \quad \text{Power Rule}$$

$$= \left[\ln(x^2) - \ln(y^{\frac{2}{3}})\right] - \ln\left[(z+1)^{\frac{1}{3}}\right]$$

$$= \ln\left(\frac{x^2}{y^{\frac{2}{3}}}\right) - \ln\left[(z+1)^{\frac{1}{3}}\right] \qquad \text{Quotient Rule}$$

$$= \ln\left[\frac{x^2}{y^{\frac{2}{3}}}{(z+1)^{\frac{1}{3}}}\right] \qquad \text{Quotient Rule}$$

$$= \boxed{\ln\left(\frac{x^2}{y^{\frac{2}{3}}(z+1)^{\frac{1}{3}}}\right)} \qquad \underline{\text{Ans}}$$