

4.3A Properties of Logarithmic Functions

A. Derivation of a Property

Consider $\log_b M + \log_b N$:

Let $\log_b M = x$ and $\log_b N = y$

Apply $b^{\square}$ to each side: $b^{\log_b M} = b^x$ and $b^{\log_b N} = b^y$

$$M = b^x \text{ and } N = b^y$$

Thus $MN = b^x \cdot b^y = b^{x+y}$ by exponent property

$$\text{Thus } \log_b(MN) = \log_b b^{x+y} = x+y = \log_b M + \log_b N$$

$\uparrow$
by definition!

Other logarithmic properties can be found in a similar fashion: convert to exponential equations, use property of exponents, and convert back.

B. Properties

$$1. \log_b M + \log_b N = \log_b(MN)$$

Product Rule

$$2. \log_b M - \log_b N = \log_b\left(\frac{M}{N}\right)$$

Quotient Rule

$$3. p \cdot \log_b M = \log_b(M^p)$$

Power Rule

C. Difficulty Using These Properties

1. There are situations that look similar to these properties but they are not the same. You must be very careful to only use valid properties.

2. The work you show should demonstrate valid properties. Several incorrect applications of properties in sequence, but resulting by the "correct last line" is in fact completely wrong.

3. We now get some practice deciding what is a valid property and what is not.

D. Examples

In each example, two expressions are given.

Identify the expression that can be changed by a property and change it; note the other as having no property.

Example 1: A) $\log_3 7 + \log_3 x$
 B) $\log_3 (7+x)$

Solution

A) $\log_3 (7x)$ by product rule

B) no property

Example 2: A) $\frac{\log_7 x}{\log_7 2}$ B) $\log_7 \left(\frac{x}{2}\right)$

Solution

A) no property

B) $\log_7 x - \log_7 2$ by quotient rule (backward)

Example 3: A) $(\log_5 x)^3$

B) $\log_5(x^3)$

Solution

A) no property

B) $3\log_5 x$ by power rule (backward)

Example 4: A) $(\log_2 5)(\log_2 x)$

B) $\log_2(5x)$

Solution

A) no property

B) $\log_2 5 + \log_2 x$ by product rule (backward)

Example 5: A) $\ln y - \ln z$

B) $\ln(y-z)$

Solution

A) $\ln\left(\frac{y}{z}\right)$ by quotient rule

B) no property

E. Comment on Property Use

The domain of the underlying logarithmic function may change upon use of these properties:

Product Rule: $\log_7(x^2) = 2\log_7 x$

$\nwarrow$ $x = -3$ works

$\nwarrow$ $x = -3$ does not work