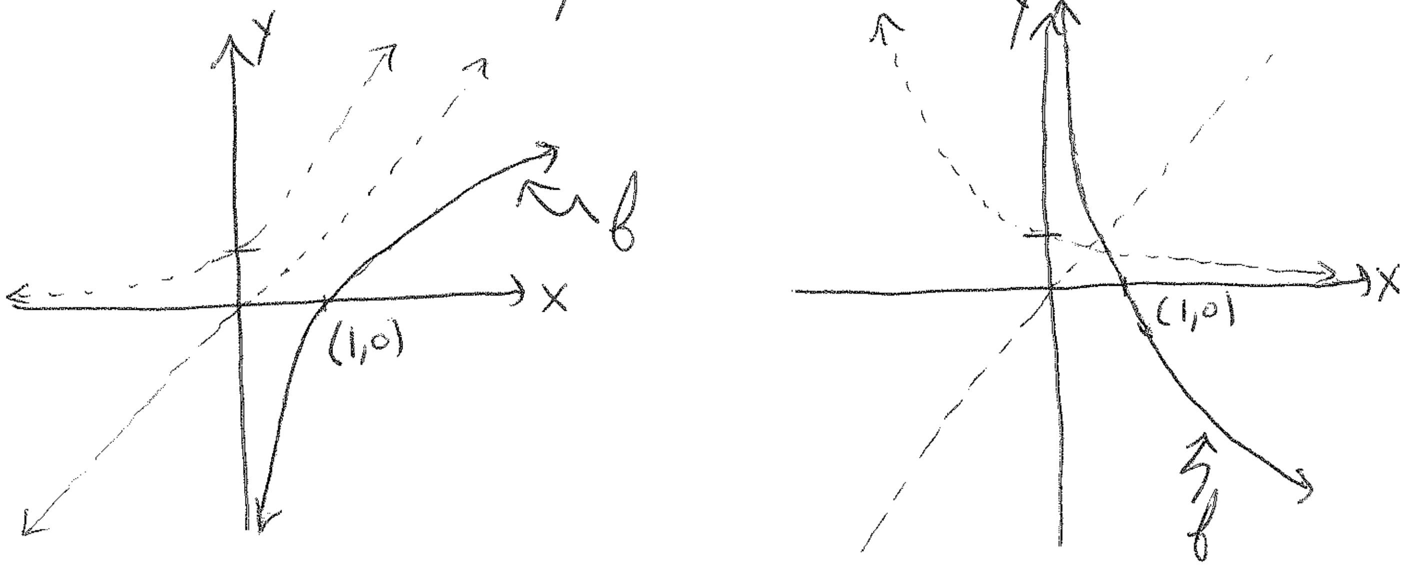
4.2B Graphs of Logarithmic Functions and Doma

A. Introduction: Base Graphs

The graphs of the logarithmic functions can be obtained by reflecting the graphs of the exponential functions across the $y=x$ line



$b>1$ $\qquad$ $0<b<1$

$$f(x)=\log_b x$$

dom $f=(0,\infty)$

rng $f=(-\infty,\infty)$

(1,0) is on the graph

f has the <u>y-axis</u> as a <u>vertical asymptote</u>

B. Domain

Since dom $f=(0,\infty)$, we are <u>not</u> allowed to put <u>negative numbers or zero</u> into logarithmic functions. We now have a third item to add to our domain checking list,

1. Denominators: can't be zero; set $\neq 0$
2. Even Roots: can't be negative inside; set ≥ 0
3. Logarithms: can't be negative or zero inside; set >0

<u>Example1</u>: Given $f(x) = \log_3(4-x) + 2$; find dom f

<u>Solution</u>

Logarithm: set inside >0

$$4 - x > 0 \Rightarrow -x > -4 \Rightarrow x < 4$$

<u>Ans</u>: $\boxed{\text{dom } f = (-\infty, -4)}$

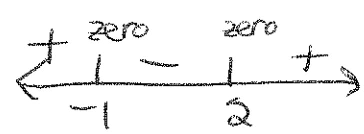
<u>Example2</u>: Given $f(x) = \log_7\left((x+1)(x-2)\right)$. Find dom f

<u>Solution</u>

Logarithm: set inside >0

$$(x+1)(x-2) > 0$$

Breakpoints: $-1, 2$



<u>Ans</u>: $\boxed{\text{dom } f = (-\infty, -1) \cup (2, \infty)}$

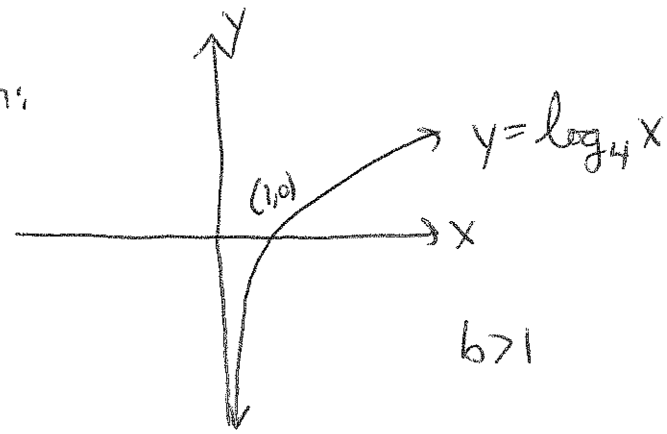
C. Graphing Logarithmic Functions

As before, we identify the base graph, perform HSRV, and keep in mind how (1,0) and the vertical asymptote move.

Example 1: Graph f, where $f(x) = \log_4(x+1) - 2$

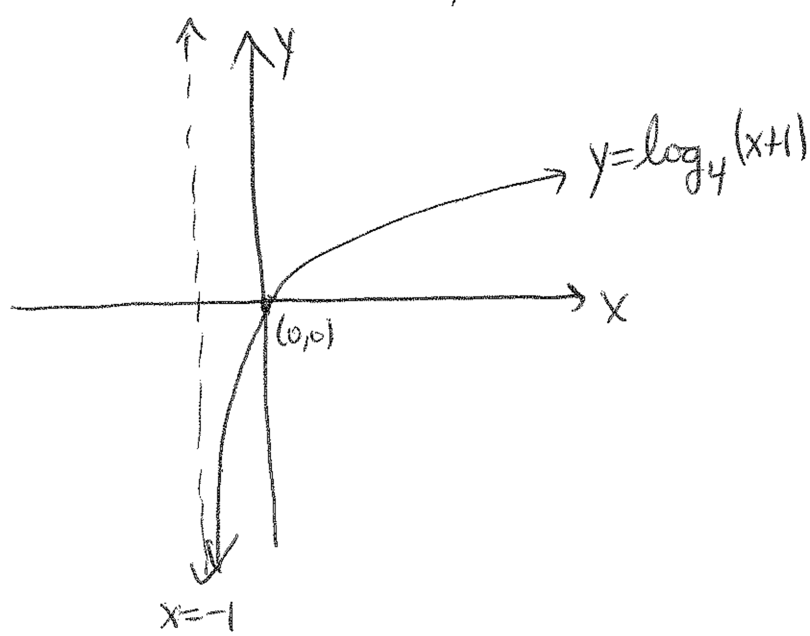
Solution

Base Graph:



H: move left 1: $(1,0) \to (0,0)$

vertical asymptote moves left 1

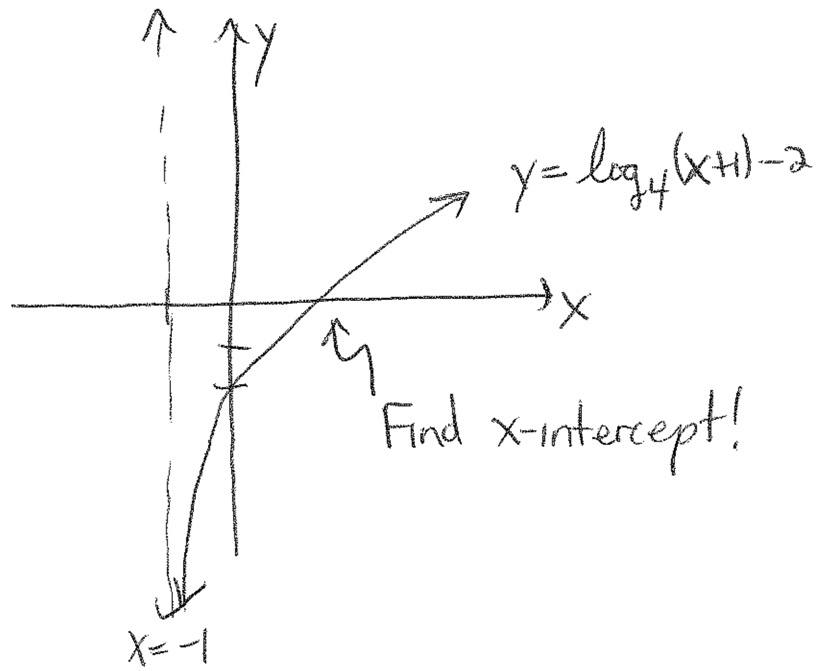


S: none

R: none

V: move down 2: $(0,0) \longrightarrow (0,-2)$

vertical asymptote does not move



x-intercept: set $y = 0$: $\quad 0 = \log_4(x+1) - 2$

$$\log_4(x+1) = 2$$

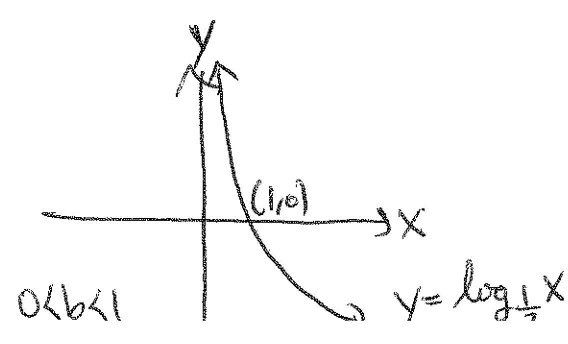
Apply $4^{\square}$ to each side: $\cancel{4^{\log_4}}{}^{(x+1)} = 4^2$

$$x+1 = 16 \implies x = 15!$$

Example 2: Graph g, where $g(x) = -4\log_{\frac{1}{7}}(3-x) + 1$

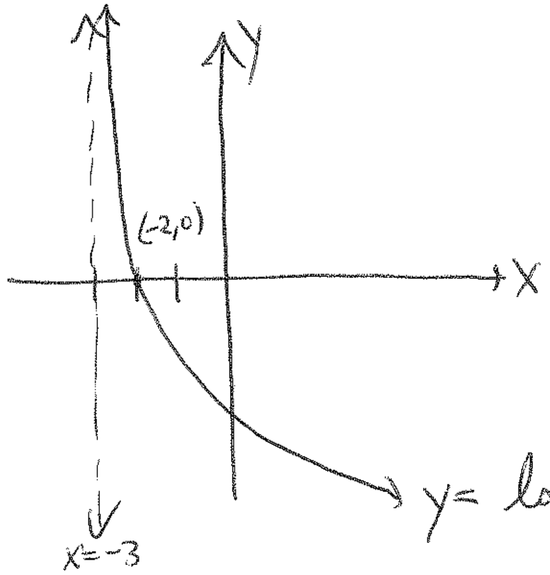
Solution

Base Graph:



H: move left 3 : $(1,0) \to (-2,0)$

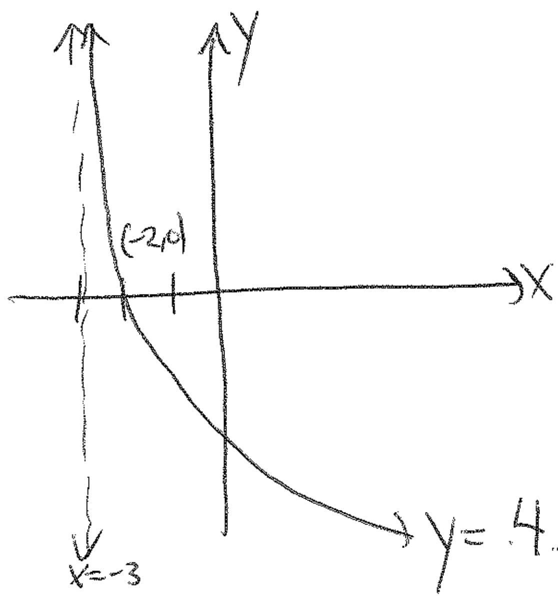
asymptote moves 3 to left



$y = \log_{\frac{1}{7}}(x+3) = \log_{\frac{1}{7}}(3+x)$

S: stretch by factor of 4 : $(-2,0) \to (-2,\underset{4\cdot 0=0}{0})$

asymptote does not change

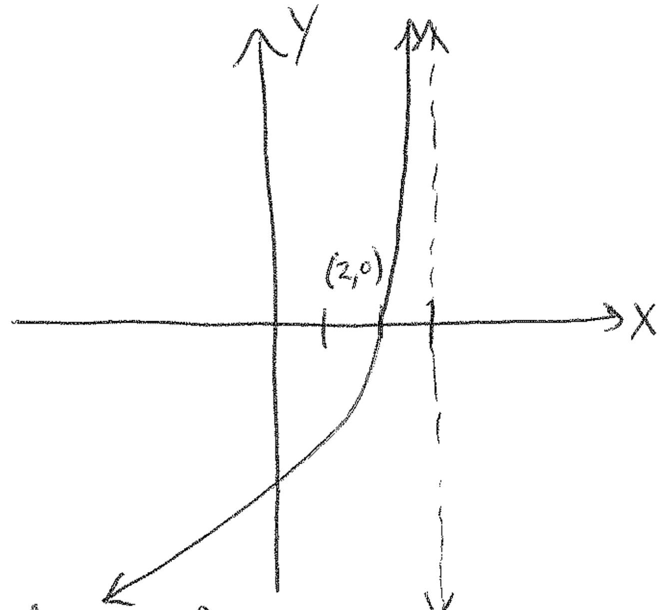


$y = 4 \log_{\frac{1}{7}}(3+x)$

R: two reflections!!

R1: across y-axis: $(-2,0) \to (2,0)$

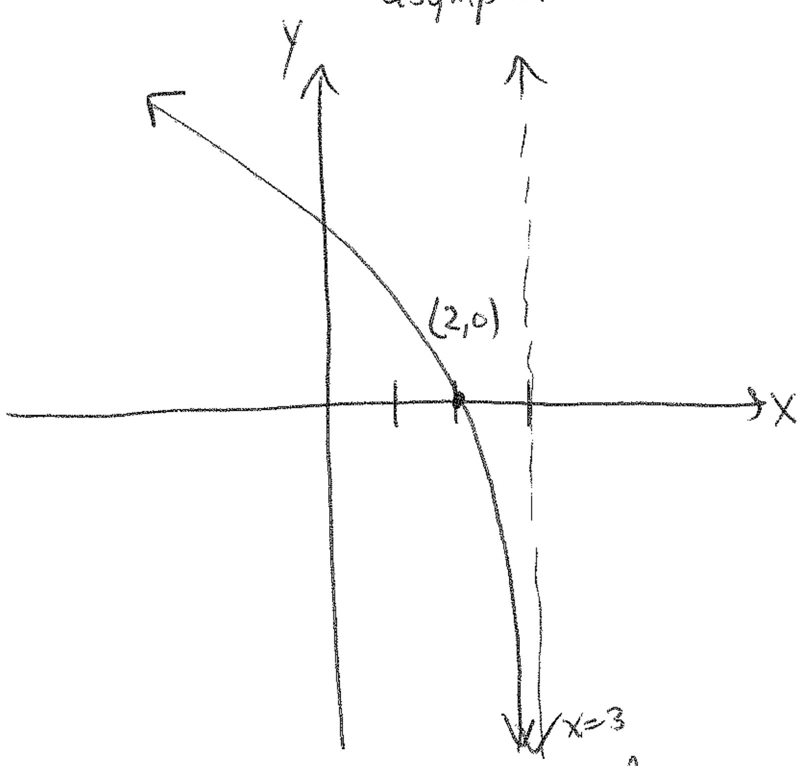
$x=-3$ asymptote $\Rightarrow x=3$ asymptote



$y = 4\log_{\frac{1}{7}}(3-x)$

R2: across x-axis: $(2,0)$ does not move

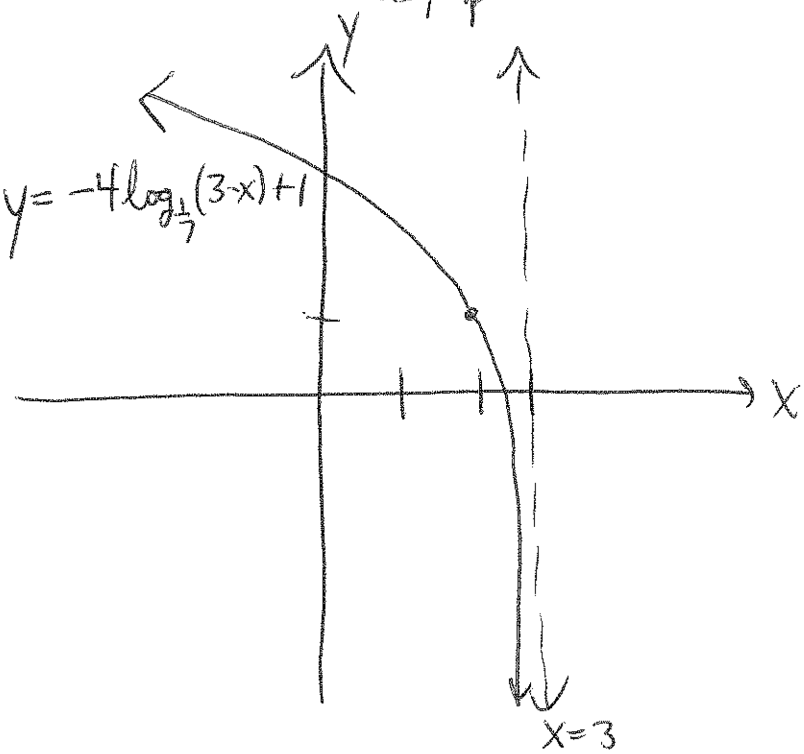
asymptote does not move



$y = -4\log_{\frac{1}{7}}(3-x)$

V: move up 1 : $(2,0) \to (2,1)$

asymptote does not move



Find x & y intercepts:

<u>x-intercept</u>: set $y=0$: $0 = -4\log_{\frac{1}{7}}(3-x)+1$

$$4\log_{\frac{1}{7}}(3-x) = 1$$

$$\log_{\frac{1}{7}}(3-x) = \frac{1}{4}$$

Apply $\left(\frac{1}{7}\right)^{\square}$ to each side: $\cancel{\tfrac{1}{7}}^{\log_{\frac{1}{7}}(3-x)} = \left(\frac{1}{7}\right)^{1/4}$

$$3-x = \left(\frac{1}{7}\right)^{1/4} \Rightarrow x = 3-\left(\frac{1}{7}\right)^{1/4}$$

$$\approx 2.9996$$

<u>y-intercept</u>: set $x=0$: $y = -4\log_{\frac{1}{7}} 3 + 1$

$$\approx 3.258$$

How do you evaluate $\log_{\frac{1}{7}}$ on a calculator? Stay tuned.