4.1 B Special Exponential Equations

A. Base Property

Since exponential functions are one-to-one, by the formal method

$f(x) = f(a)$ solves to $x = a$.

Now $f(x) = b^x$, so we have the base property:

Base Property: $\boxed{b^x = b^a \text{ solves to } x = a}$

B. Solving Some Special Exponential Equations

Write each side of the equation using the same base.

Apply the base property.

C. Examples

Example 1: Solve $5^x = 125$ for x

Solution

Write each side using base 5:

$$5^x = 5^3$$

Apply Base Property: $x = 3$

Ans: $\boxed{\{3\}}$

Example 2: Solve $\left(\frac{1}{2}\right)^{2x+3} = 4$ for x

Solution

Write each side with the same base

We can use base 2!

$$\left(2^{-1}\right)^{2x+3} = 2^2$$

$$2^{-2x-3} = 2^2$$

Base Property: $-2x-3=2 \Rightarrow -2x=5$

Ans: $\boxed{\left\{ \frac{-5}{2} \right\}}$

Example 3: Solve $9^{x^2} = \left(\frac{1}{27}\right)^{3x-1}$ for x

Solution

Write each side with the same common base: 3

$$\left(3^2\right)^{x^2} = \left(3^{-3}\right)^{3x-1}$$

$$3^{2x^2} = 3^{-9x+3}$$

Base Property: $2x^2 = -9x+3$

$$2x^2+9x-3=0$$

QF: $x = \dfrac{-9 \pm \sqrt{81-4(2)(-3)}}{2(2)} = \dfrac{-9 \pm \sqrt{81+24}}{4}$

Ans: $\boxed{\left\{ \frac{-9 \pm \sqrt{105}}{4} \right\}}$