4.1A Exponential Functions

A. Definition of an Exponential Function

f is an <u>exponential function</u> if $f(x) = b^x$,

where b is a positive constant not equal to 1 (*)

i.e. $f(x) = 2^x$ and $f(x) = \left(\frac{1}{3}\right)^x$, etc. define exponential functions

<u>Reason for (*)</u>:

1. If $b<0$, we could get nonreal complex numbers which we disallow for functions:

$$f(x) = (-4)^x$$

$$\Rightarrow f\left(\tfrac{1}{2}\right) = (-4)^{\frac{1}{2}} = \sqrt{-4} = i\sqrt{4} = 2i \;\times$$

2. If $b=0$, $f(x) = 0^x \Rightarrow f(x) = 0$ [since $0^x = 0$]

but $y=0$ is a <u>horizontal line</u>

3. If $b=1$, $f(x) = 1^x \Rightarrow f(x) = 1$ [since $1^x = 1$]

but $y=1$ is a <u>horizontal line</u>.

<u>Note:</u> For exponential functions such as the one given by $f(x) = 3^x$, the variable is in the exponent spot!

This differs from a power function, i.e. $f(x) = x^3$.

B. Evaluation

Example 1: Let $f(x) = 4^x$. Find . a) $f(2)$

b) $f(\frac{1}{2})$

c) $f(-\frac{1}{2})$

Solution

a) $f(2) = 4^2 = \boxed{16}$

b) $f(\frac{1}{2}) = 4^{\frac{1}{2}} = \sqrt{4} = \boxed{2}$

c) $f(-\frac{1}{2}) = 4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{\sqrt{4}} = \boxed{\frac{1}{2}}$

Example 2: Let $f(x) = 2(\frac{1}{5})^x$. Find a) $f(2)$

b) $f(0)$

c) $f(-2)$

Solution

a) $f(2) = 2(\frac{1}{5})^2 = 2(\frac{1}{25}) = \boxed{\frac{2}{25}}$

b) $f(0) = 2(\frac{1}{5})^0 = 2 \cdot 1 = \boxed{2}$

c) $f(-2) = 2(\frac{1}{5})^{-2} = 2(5)^2 = 2 \cdot 25 = \boxed{50}$

Warning: $2(3^x)$ is <u>not</u> 6^x!

We can't multiply the number on the outside to the base.

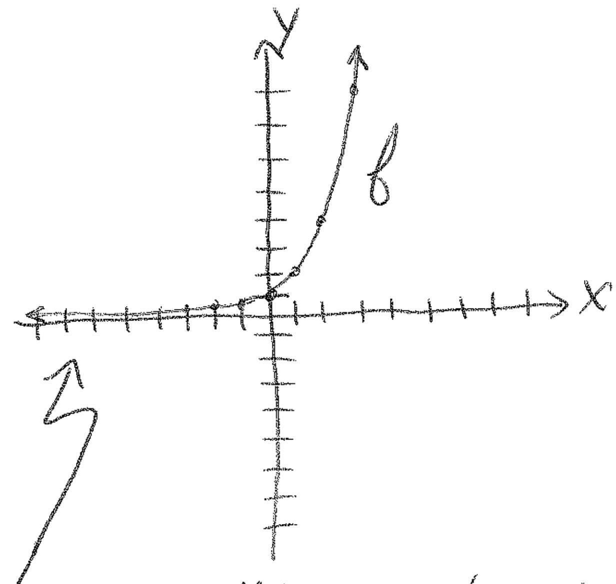
[In fact, $6^x = (2\cdot 3)^x = 2^x \cdot 3^x$]

C. Graphing Example for Motivation

Graph f, where $f(x) = 2^x$

Solution

x	y
-3	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$



as $x \to -\infty$, $f(x) \to 0$ (approaches the x-axis)

Note: $\text{dom } f = (-\infty, \infty)$

$\text{rng } f = (0, \infty)$

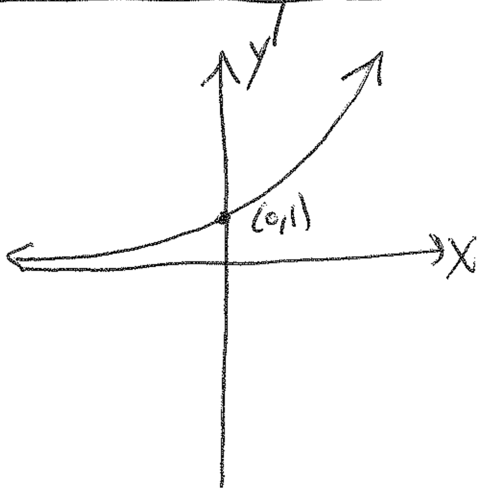
f is increasing

f is one-to-one

f has a horizontal asymptote (the x-axis)

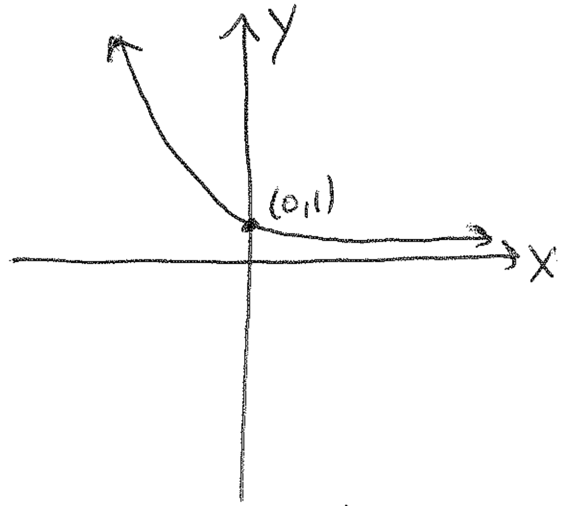
$(0,1)$ is a point on the graph

D. Summary of Graph Characteristics



$b>1$

f is increasing



$0<b<1$

f is decreasing

$\text{dom } f = (-\infty, \infty)$

$\text{rng } f = (0, \infty)$

f is one-to-one

f has the x-axis as an asymptote

$(0,1)$ is on the graph

E. Strategy for Graphing Exponential Functions

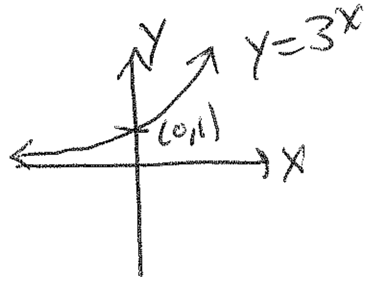
1. Determine the base graph $y=b^x$ for $b>1$ or $0<b<1$
2. Perform HSRV transformations to get the desired graph.
3. Make sure to relocate the point $(0,1)$ and the horizontal asymptote. Use these as reference "points".

F. Graphing Examples

Example 1: Graph f, where $f(x) = 3^{x+1} - 2$

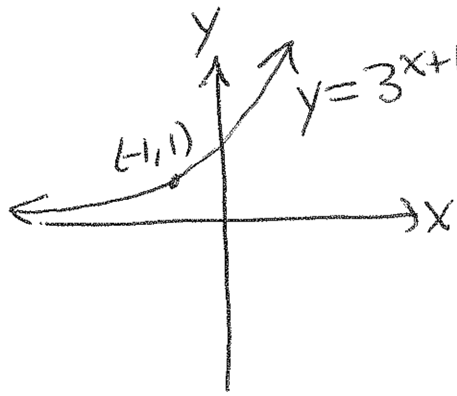
Solution

Base Graph:

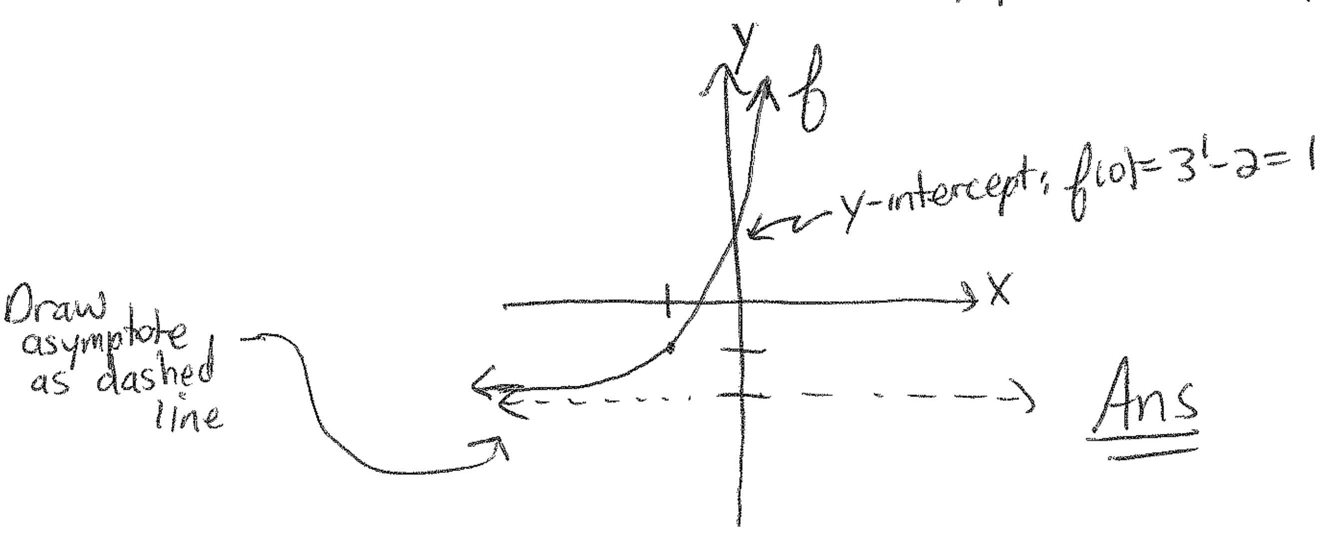


Now use HSRV!

H: move left by 1: $(0,1)$ point $\Rightarrow (-1,1)$
asymptote does not move



S: none
R: none
V: move down 2: "$(0,1)$ point" $(-1,1) \Rightarrow (-1,-1)$
asymptote $\Rightarrow$ drops down 2

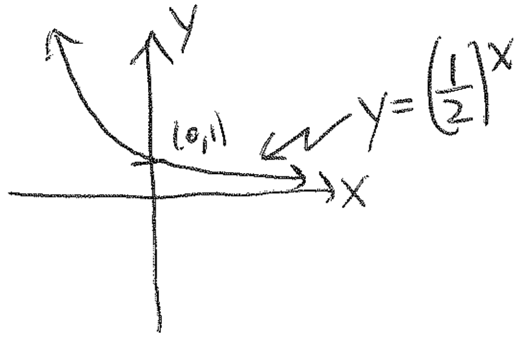


Ans

Example 2: Graph f, where $f(x) = 3 - 2\left(\frac{1}{2}\right)^{x-2}$

Solution

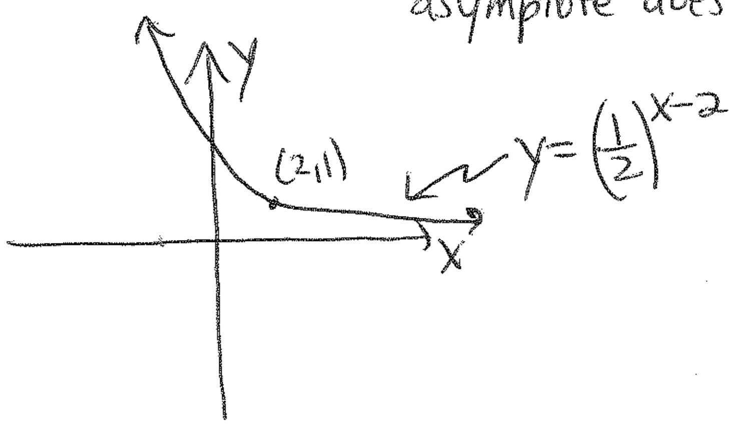
Base Graph:



Now use HSRV!

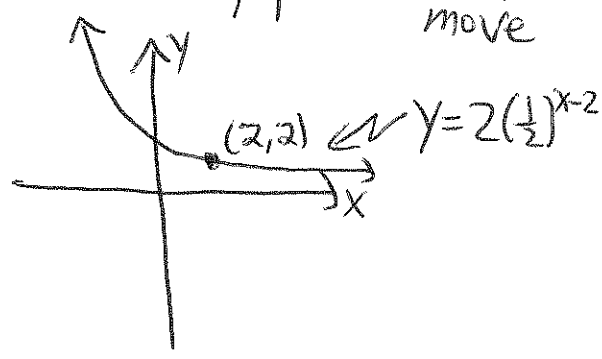
H: move right 2: $(0,1) \Rightarrow (2,1)$

asymptote does not move



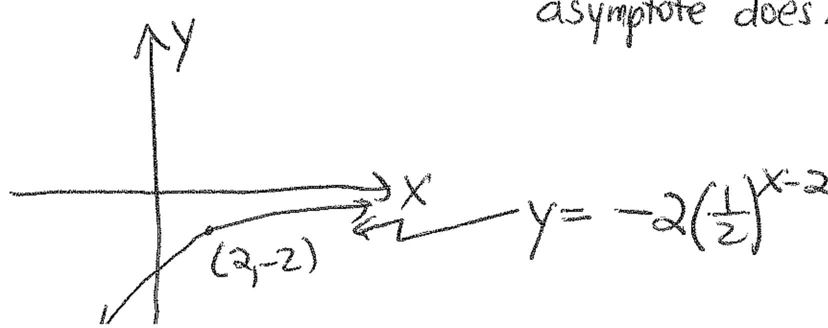
S: stretch by factor of 2: $(2,1) \Rightarrow (2,2)$

asymptote does not move



R: reflect across x-axis $(2,2) \Rightarrow (2,-2)$

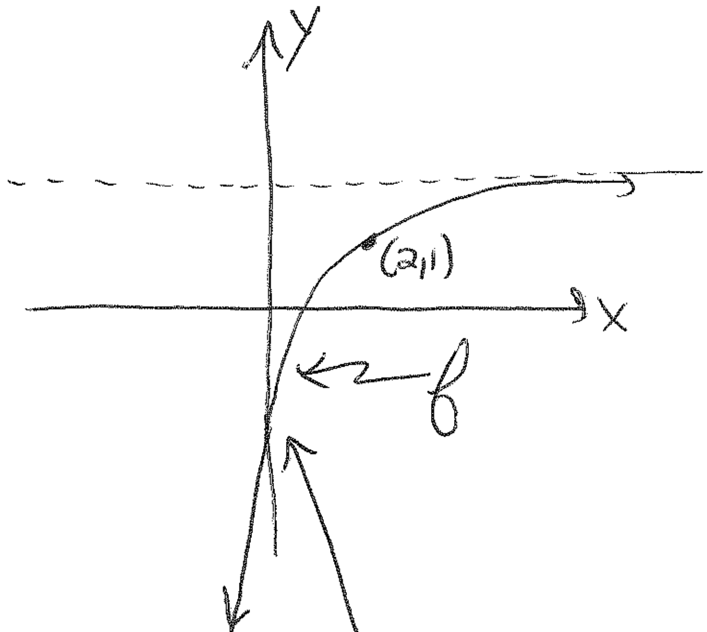
asymptote does not move



V: up 3

$(2,-2) \Rightarrow (2,1)$

asymptote moves up 3



y-intercept: $f(0) = 3 - 2\left(\frac{1}{2}\right)^{-2}$

$= 3 - 2(2)^2$

$= 3 - 8 = -5$

Ans