3.6B Rational Inequalities

A. Introduction

As in the polynomial inequality case, we must move everything to one side, factor, and use the breakpoint method

Note: You are NOT ALLOWED to clear fractions (Inequalities can't handle it!)

B. New Features

1. Breakpoints in the numerator are zero breakpoints (as in the last section)

2. However, breakpoints in the denominator are undefined breakpoints.

C. Examples

Example 1: Solve $\dfrac{x^2+x-3}{x^2-x-12} \geq 0$ for x

Solution

Factor:

x^2+x-3	-3 $+,-$
$x^2+2x-x-3$	-2
$x^2+3x-2x-3$	-6 X

prime

x^2-x-12	-12 $+,-$
$x^2+x-2x-12$	-2
$x^2+2x-3x-12$	-6
$x^2+3x-4x-12$	-12 ✓

$x(x+3)-4(x+3)$

$(x+3)(x-4)$

We have $\dfrac{x^2+x-3}{(x+3)(x-4)} \geq 0 \qquad (*)$

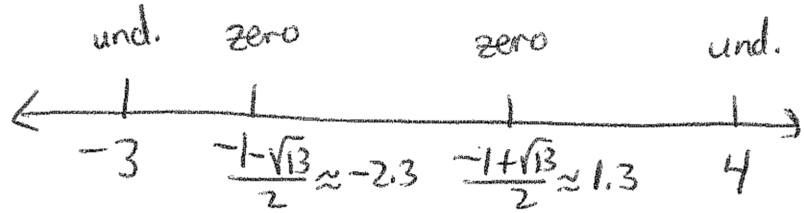
Now use strategy:

1. <u>Breakpoints</u>: $x^2+x-3=0$ prime, so use QF

$$x = \frac{-1 \pm \sqrt{1^2-4(1)(-3)}}{2} = \frac{-1 \pm \sqrt{13}}{2}$$

$$x+3=0 \Rightarrow x=-3$$
$$x-4=0 \Rightarrow x=4$$

2. Distinguish breakpoint "type" on number line:



3. Test zones using $(*)$:

<u>Test I</u>: Pick $x=-4$: $\dfrac{(-4)^2+(-4)-3}{(-4+3)(-4-4)}$

$$= \frac{+}{(-)(-)} = +$$

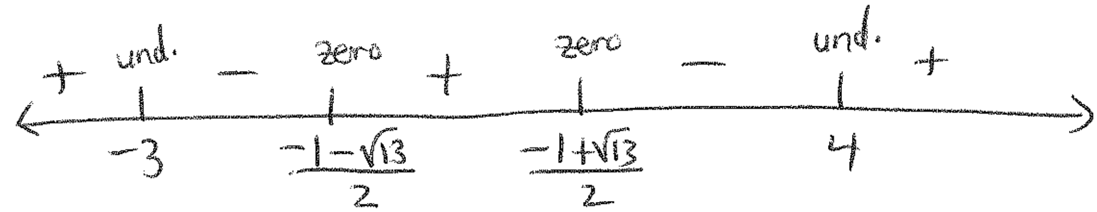
<u>Test II</u>: Pick $x=-2.5=\frac{-5}{2}$: $\dfrac{\left(\frac{-5}{2}\right)^2+\left(\frac{-5}{2}\right)-3}{\left(\frac{-5}{2}+3\right)\left(\frac{-5}{2}-4\right)}$

$$= \frac{\frac{25}{4}-\frac{10}{4}-\frac{12}{4}}{\left(\frac{1}{2}\right)\left(\frac{-13}{2}\right)} = \frac{+}{-} = -$$

Test III: Pick $x=0$: $\dfrac{0^2+0-3}{(0+3)(0-4)} = \dfrac{-}{(+)(-)} = \dfrac{-}{-} = +$

Test IV: Pick $x=2$: $\dfrac{2^2+2-3}{(2+3)(2-4)} = \dfrac{+}{(+)(-)} = -$

Test V: Pick $x=5$: $\dfrac{5^2+5-3}{(5+3)(5-4)} = \dfrac{+}{(+)(+)} = +$

4. 

By (*), $\dfrac{x^2+x-3}{(x+3)(x-4)} \geq 0$, we want positive or zero.

Ans: $\boxed{(-\infty,-3) \cup \left[\frac{-1-\sqrt{13}}{2}, \frac{-1+\sqrt{13}}{2}\right] \cup (4,\infty)}$

Example 2: Solve $\dfrac{(x-4)^2(x+3)}{3x^2+5x-1} \leq 0$ for x

Solution

Everything is already factored since $3x^2+5x-1$ is prime

Use strategy

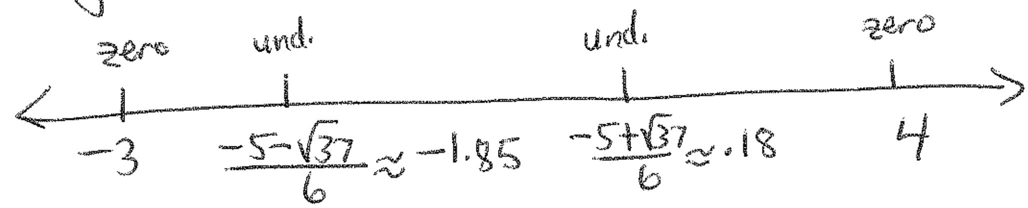
1. Find breakpoints: $(x-4)^2=0 \Rightarrow x=4$

$x+3=0 \Rightarrow x=-3$

$3x^2+5x-1=0$ prime, so use QF

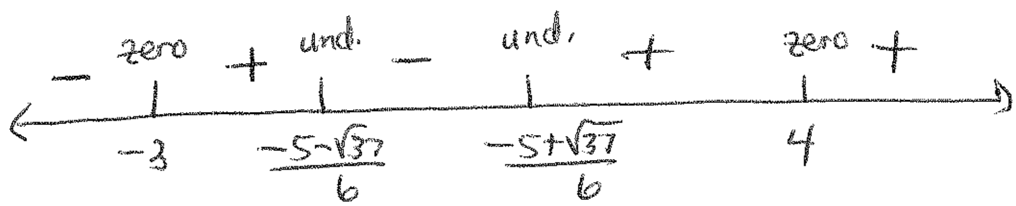
$$x = \frac{-5 \pm \sqrt{25-4(3)(-1)}}{2(3)} = \frac{-5 \pm \sqrt{37}}{6}$$

2. Distinguish breakpoint "type" on number line:



3. Test zones

(details omitted)



<u>note</u>: does not alternat

4. Since we have $\frac{(x-4)^2(x+3)}{3x^2+5x-1} \leq 0$, we want

<u>negative</u> or <u>zero</u>

<u>Ans</u>:

$$\boxed{(-\infty, -3] \cup \left(\frac{-5-\sqrt{37}}{6}, \frac{-5+\sqrt{37}}{6}\right) \cup \{4\}}$$

$$\text{-- or --}$$

$$(-\infty, -3] \cup \left(\frac{-5-\sqrt{37}}{6}, \frac{-5+\sqrt{37}}{6}\right) \cup [4,4]$$

Example 3: Solve $\frac{5}{x} \le 2$ for x

Solution

Warning: Not allowed to clear fractions

Move to one side, simplify, and have factored:

$$\frac{5}{x} - 2 \le 0$$

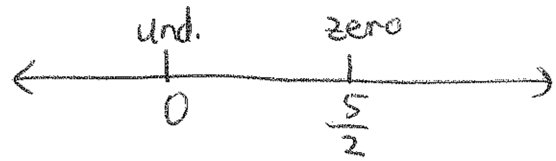
$$\frac{5}{x} - \frac{2x}{x} \le 0$$

$$\frac{5-2x}{x} \le 0 \quad (*)$$

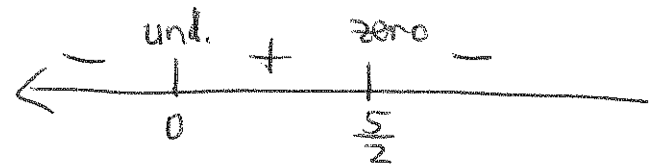
1. **Breakpoints:** $5-2x=0 \Rightarrow 2x=5 \Rightarrow x=\frac{5}{2}$

   $x=0$

2. **Number Line:**



3. **Test zones using (*) and label line:**



4. **Ans:** $\boxed{(-\infty,0) \cup \left[\frac{5}{2},\infty\right)}$