

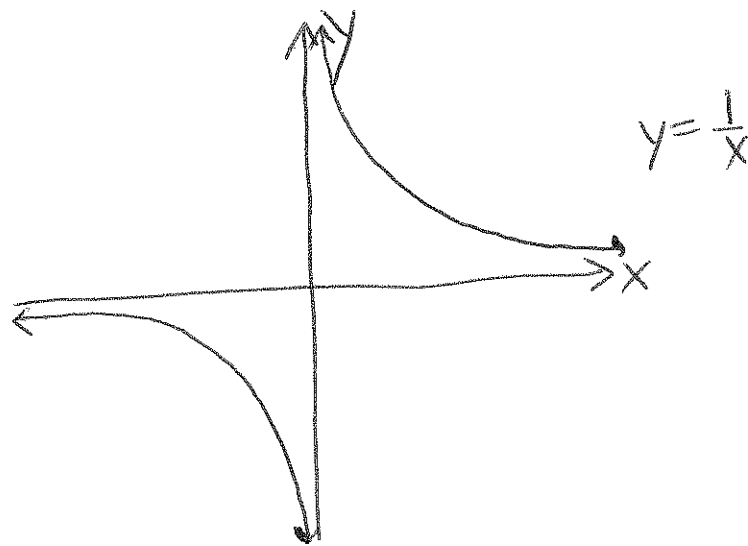
3.5B Mathematical Description of Asymptotes

A. Introduction

To describe the behavior of asymptotes mathematically, we use the arrow notation from the mathematical end behavior of polynomials

We not only ask for what happens to $f(x)$ as $x \rightarrow -\infty$ or $x \rightarrow \infty$ as in the polynomial case, but we also ask what happens when x approaches a fixed number.

Consider



We see that as $x \rightarrow -\infty$, $f(x) \rightarrow 0$
as $x \rightarrow \infty$, $f(x) \rightarrow 0$

However, what does $f(x) \rightarrow ?$, when x approaches 0?

Clearly, it matters which way we approach 0!

If we approach 0 from the right side, $f(x) \rightarrow \infty$

If we approach 0 from the left side, $f(x) \rightarrow -\infty$

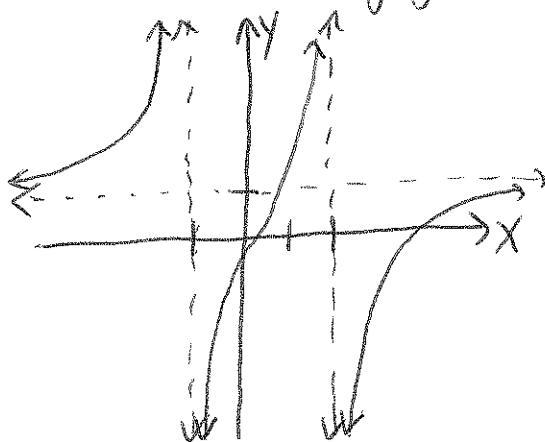
We write $x \rightarrow 0^+$ for x approaching 0 from the right.
We write $x \rightarrow 0^-$ for x approaching 0 from the left.

B. Arrow Notation

1. $x \rightarrow \infty$ means x goes to infinity
2. $x \rightarrow -\infty$ means x goes to negative infinity
3. $x \rightarrow a^+$ means x approaches " a " from the right
4. $x \rightarrow a^-$ means x approaches " a " from the left

C. Example

Mathematically describe the asymptote behavior using arrow notation for the following graph of f :



Solution

as $x \rightarrow -\infty$, $f(x) \rightarrow 1$	;	as $x \rightarrow 2^-$, $f(x) \rightarrow \infty$
as $x \rightarrow -1^-$, $f(x) \rightarrow \infty$	;	as $x \rightarrow 2^+$, $f(x) \rightarrow -\infty$
as $x \rightarrow -1^+$, $f(x) \rightarrow -\infty$	;	as $x \rightarrow \infty$, $f(x) \rightarrow 1$