3.5A Reciprocal Functions

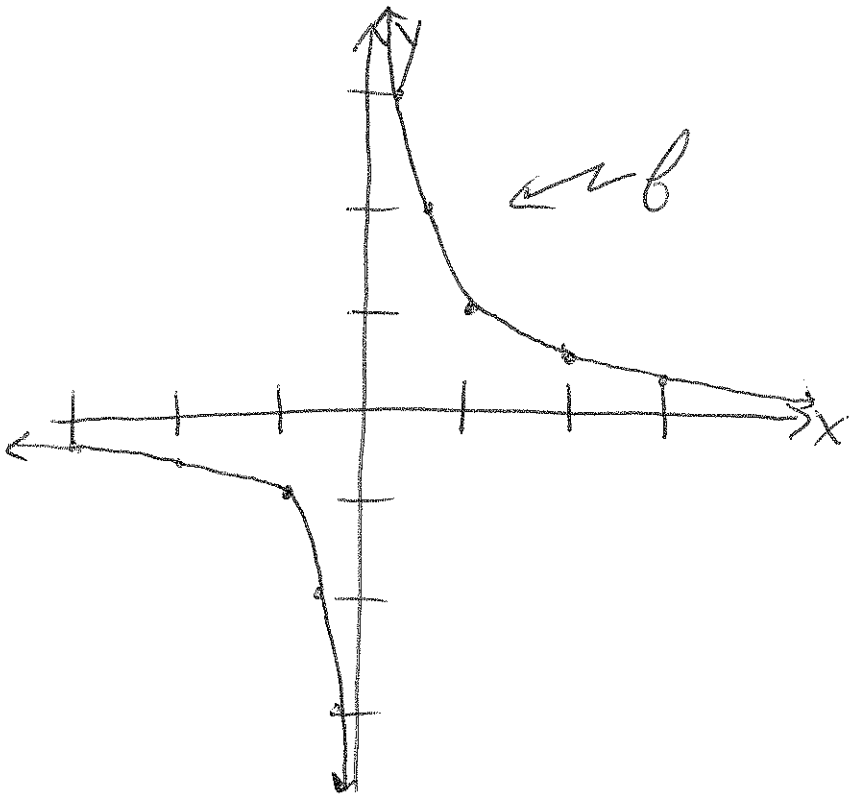
A. Graphing the Reciprocal Function

Let $f(x)=\frac{1}{x}$. f is called the reciprocal function.

We plot by making a table of values

Since f is undefined at $x=0$, we pick a lot of points near 0.

X	Y
-10	$-\frac{1}{10}$
-3	$-\frac{1}{3}$
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
$-\frac{1}{3}$	-3
$-\frac{1}{10}$	-10
0	und.
$\frac{1}{10}$	10
$\frac{1}{3}$	3
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$
10	$\frac{1}{10}$



Note: For very large numbers, i.e. 10000 and very negative numbers, i.e. −10000, the graph approaches the x-axis. For numbers near zero, the graph approaches, but does not cross the y-axis.

Imaginary approaching lines are called asymptotes.

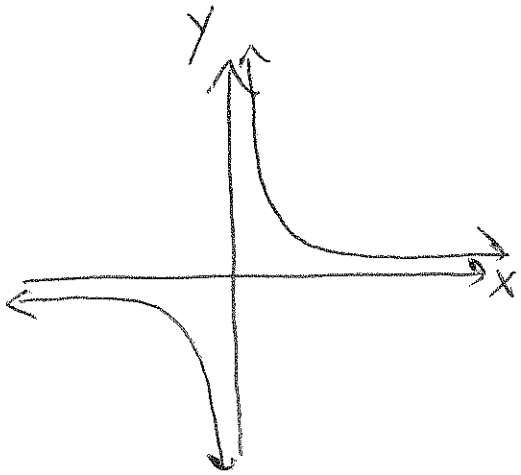
B. Transformations of the Reciprocal Function

We graph transformations by applying HSRV to the base graph given by $f(x)=\frac{1}{x}$. We must make sure to move the asymptotes. We draw in the asymptotes as dashed lines. Find intercepts!

Example1: Graph f, where $f(x)=\frac{1}{x-2}+3$

Solution

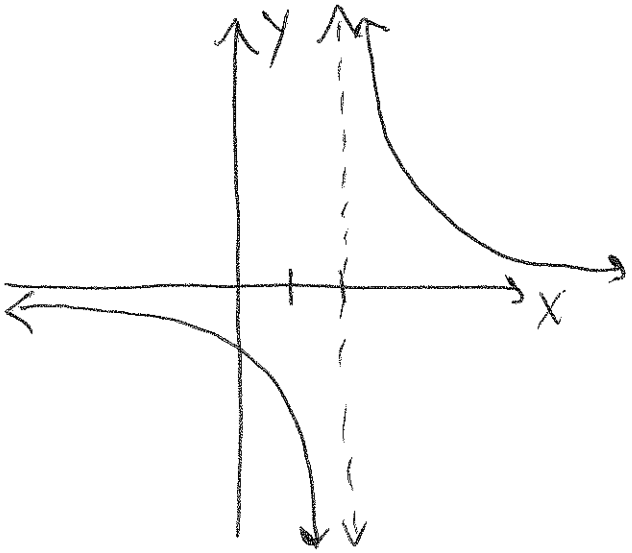
Base Graph:



H: subtract 2 inside: move right 2

horizontal asymptote stays

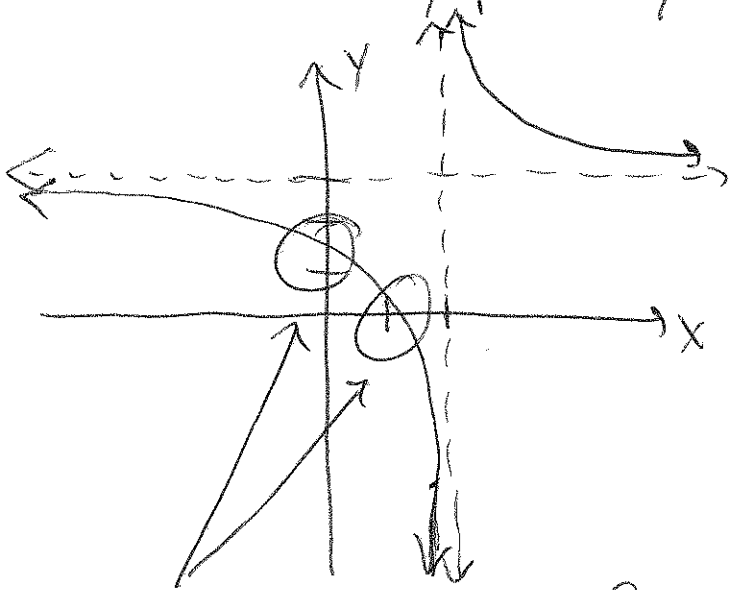
vertical asymptote moves right 2



S: none

R: none

V: up 3 : horizontal asymptote moves up 3
vertical asymptote stays



need x & y intercepts for accuracy!

x-intercept: set $y=0$

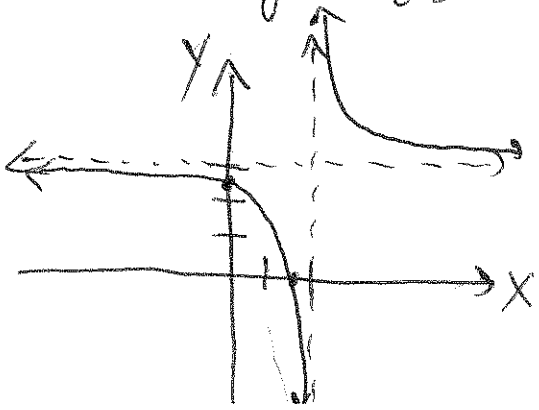
$$\frac{1}{x-2}+3=0$$

Disallowed: $x \neq 2$
LCD $= x-2$
Multiply by LCD

$$1+3(x-2)=0$$
$$1+3x-6=0$$
$$3x-5=0 \Rightarrow x=\frac{5}{3}$$

y-intercept: set $x=0$: $f(0)=\frac{1}{0-2}+3=-\frac{1}{2}+3=\frac{5}{2}$

Final Answer:

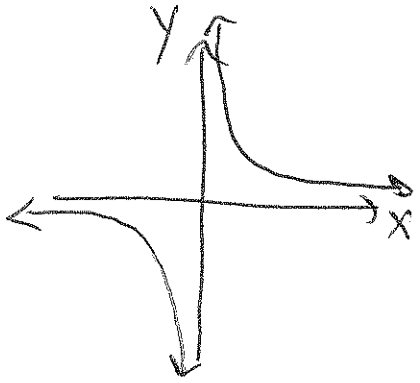


Example 2: Graph f, where $f(x) = \frac{-3}{x+1} - 2$

Also find dom f and rng f

Solution

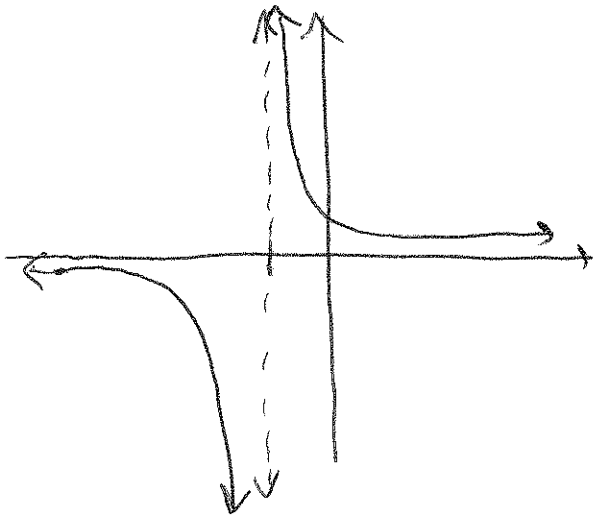
Base Graph:



$\text{dom } f_b = (-\infty, 0) \cup (0, \infty)$

$\text{rng } f_b = (-\infty, 0) \cup (0, \infty)$

H: add 1 inside : left 1 : horizontal asymptote stays
vertical asymptote moves

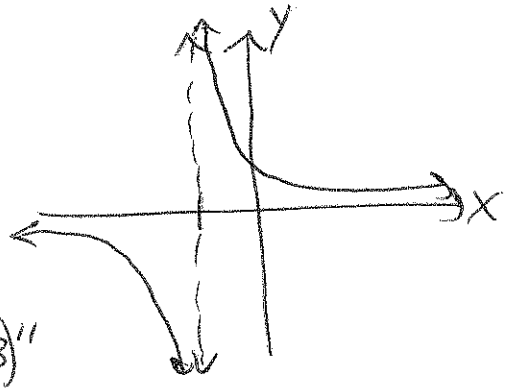


$\text{dom } f_h =$ "$(-\infty - 1, 0 - 1) \cup (0 - 1, \infty - 1)$" X-values change

$= (-\infty, -1) \cup (-1, \infty)$

$\text{rng } f_h = (-\infty, 0) \cup (0, \infty)$ y-values fixed

S: stretch y-values by 3
asymptotes unchanged



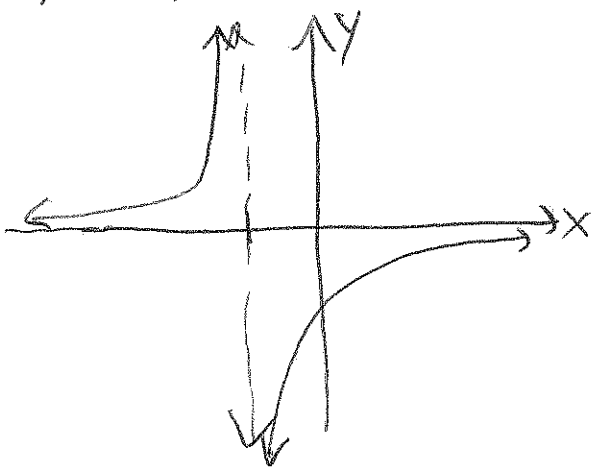
$\text{dom } f_S = (-\infty,-1)\cup(-1,\infty)$

$\text{rng } f_S = "(-\infty\cdot 3, 0\cdot 3)\cup(0\cdot 3,\infty\cdot 3)"$
$(-\infty,0)\cup(0,\infty)$

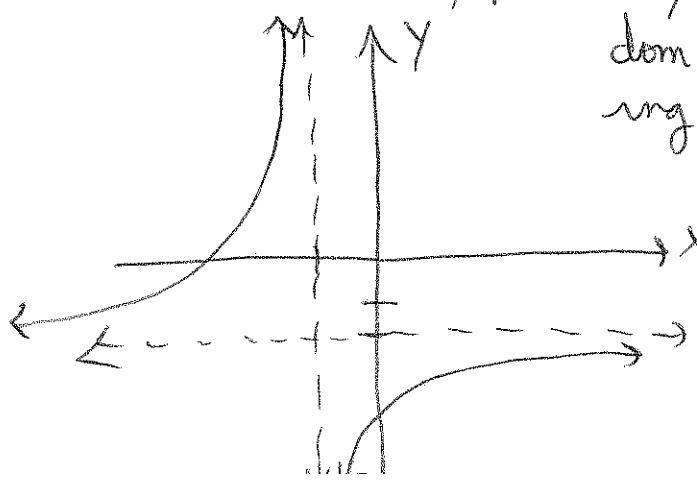
R: minus outside: reflect x-axis: x-values fixed
y-values times -1
asymptotes unchanged

$\text{dom } f_R = (-\infty,-1)\cup(-1,\infty)$

$\text{rng } f_R = "(-\infty\cdot -1, 0\cdot(-1))\cup(0\cdot(-1),\infty\cdot(-1))"$
$"(\infty,0)\cup(0,-\infty)" = "(0,\infty)\cup(-\infty,0)"$
$(-\infty,0)\cup(0,\infty)$



V: down 2: horizontal asymptote moves
vertical asymptote stays



$\text{dom } f_V = (-\infty,-1)\cup(-1,\infty)$

$\text{rng } f_V = "(-\infty-2, 0-2)\cup(0-2,\infty-2)$
$= (-\infty,-2)\cup(-2,\infty)$

Now fix intercepts!

Intercepts:

x-intercepts: set $y=0$: $\frac{-3}{x+1}-2=0$

disallowed: $x \neq -1$

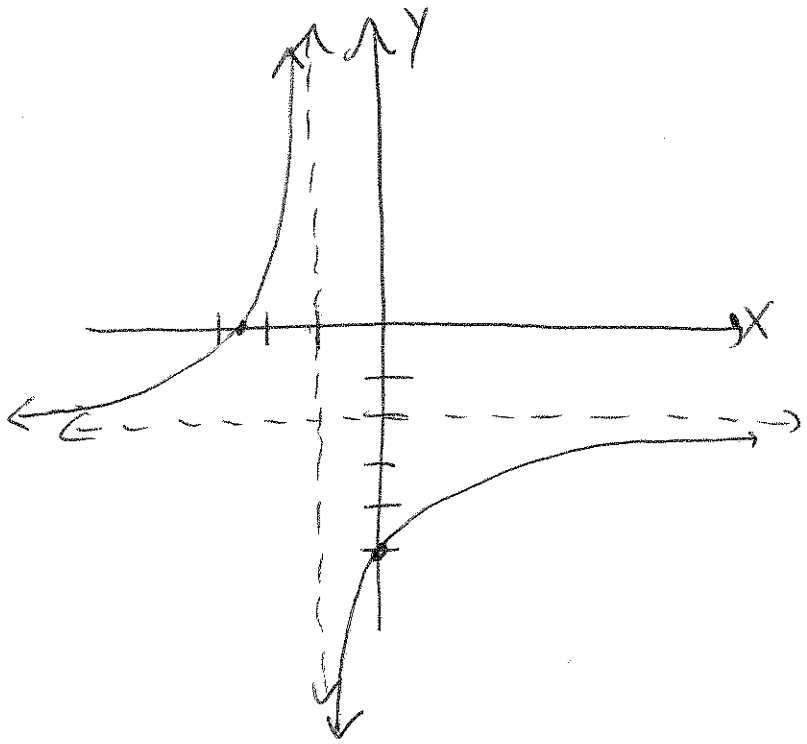
LCD $= x+1$

Multiply by LCD

$$-3-2(x+1)=0$$
$$-3-2x-2=0$$
$$2x=-5$$
$$x=\frac{-5}{2}$$

y-intercept: set $x=0$: $f(0)=\frac{-3}{0+1}-2=-3-2=-5$

Final Graph:



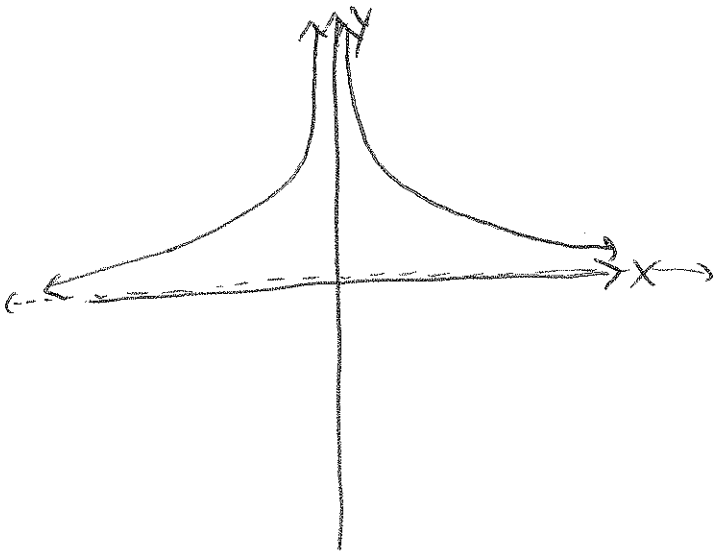
Also, $\boxed{\begin{array}{l}\text{dom } f=(-\infty,-1)\cup(-1,\infty)\\ \text{rng } f=(-\infty,-2)\cup(-2,\infty)\end{array}}$

C. Another Base Graph

Let $f(x) = \frac{1}{x^2}$.

As in the reciprocal function case, f is undefined at $x=0$.

By making a table of values, we get the following graph:



The x-axis is a horizontal asymptote and the y-axis is a vertical asymptote.

We can graph functions like g, where $g(x) = \frac{1}{(x-2)^2} + 3$ by doing HSRV transformations of the above base graph, just as we did for the reciprocal function.