3.4 B II Addendum: Descartes' Rule of Signs

A. Goal

Considering rational and irrational zeros together, we'd like to know how many real zeros a polynomial function f has.

B. Descartes' Rule of Signs

Given a polynomial $f(x)$ with nonzero constant term ($a_0 \neq 0$)

1. Let s_+ = # of sign variations in $f(x)$.
2. The number of positive real zeros is one of the numbers in the list:

$$0, 2, 4, \dots, s_+ \qquad (\text{if } s_+ \text{ is even})$$

OR

$$1, 3, 5, \dots, s_+ \qquad (\text{if } s_+ \text{ is odd})$$

Note: The number of negative real zeros is given by the same recipe, except you use s_- = # of sign variations in $f(-x)$ instead.

C. Examples

Example 1: Let $f(x) = 2x^5 - x^4 + 3x^3 - 5x^2 + 7$

Determine the number of possible real zeros (positive and negative) by Descartes' Rule of Signs.

Solution

$$f(x) = 2x^5 \overset{1}{-} x^4 \overset{2}{+} 3x^3 \overset{3}{-} 5x^2 \overset{4}{+} 7$$, so $S_+ = 4$

Thus the number of positive real zeros is 0, 2, or 4.

$f(-x) = -2x^5 - x^4 - 3x^3 - 5x^2 + 7$, so $S_- = 1$

Thus the number of negative real zeros is 1.

Example 2: Let $f(x) = 2x^5 + x^4 + 3x^3 + 5x^2 + 7$

Determine the number of possible real zeros (positive and negative) by Descartes' Rule of Signs

Solution

$f(x) = 2x^5 + x^4 + 3x^3 + 5x^2 + 7$, so $S_+ = 0$.

Thus the number of positive real zeros is 0.

$f(-x) = -2x^5 + x^4 - 3x^3 + 5x^2 + 7$, so $S_- = 3$

Thus the number of negative real zeros is 1 or 3.