3.3C Division Algorithm

A. Introduction

We seek a symbolic representation of the division process. From doing algebraic long division,

$$\frac{3x^3+4x^2+x+7}{x^2+1} = 3x+4 + \frac{-2x+3}{x^2+1}$$

Writing symbolically,

1. dividend $\longrightarrow$ $\dfrac{f(x)}{d(x)} = \underset{\substack{\uparrow \\ \text{quotient}}}{q(x)} + \dfrac{r(x)}{d(x)}$ $\begin{matrix} \leftarrow \text{remainder} \\ \leftarrow \text{divisor} \end{matrix}$

   $\nearrow$ divisor

2. $\deg r(x) < \deg d(x)$ ⇐ degree of remainder is always less than the degree of the divisor.

Multiplying both sides of the above equation by d(x) to clear fractions we get:

$$f(x) = d(x)q(x) + r(x)$$

B. Division Algorithm for Long Division

If you divide $f(x)$ by $d(x)$, the following equation holds:

$$\boxed{f(x) = d(x)q(x) + r(x)}$$

Division Algorithm Equation

where $q(x)$ is the unique quotient polynomial

and $r(x)$ is the unique remainder polynomial

and $\deg r(x) < \deg d(x)$

C. Division Algorithm for Synthetic Division

Here we have divisors that are monic linear polynomials,

so $d(x) = x - c$

Also, since $\deg r(x) < \deg d(x)$, the remainder must be a number, which we call r.

Thus the Division Algorithm Equation becomes:

$$\boxed{f(x) = (x-c)q(x) + r}$$

Division Algorithm Equation
for Synthetic Division