# 3.3B Synthetic Division

## A. Linear Polynomials

A linear polynomial is a polynomial of the form $ax+b$, $a \neq 0$

Thus $3x-1$, $2x-5$, $\frac{3}{5}x-\frac{1}{2}$ are linear polynomials.

## B. Monic Polynomials

A monic polynomial is a polynomial with leading coefficient $a_n = 1$

Thus $x^3+5x^2-2x-1$, $x^5-\frac{7}{3}x^4-2$, $x+6$ are monic polynomials

## C. Monic Linear Polynomials

A monic linear polynomial is a polynomial both linear and monic

Thus $x+5$, $x-2$, $x+\frac{17}{5}$ are monic linear polynomials.

Note: All monic linear polynomials look like

$$x - c$$

where $c$ is positive or negative [or even zero]

(ie. $x+5 \Leftrightarrow c=-5$)

## D. Long Division Example with Monic Linear Divisor

Divide $2x^4 - x^2 + 3x + 4$ by $x+2$:

$$
\begin{array}{r l}
 & 2x^3 - 4x^2 + 7x - 11 \\
x+2 \,\big)\!\!\! & 2x^4 + 0x^3 - x^2 + 3x + 4 \\
 & \underline{-(2x^4 + 4x^3)} \\
 & \quad -4x^3 - x^2 \\
 & \quad \underline{-(-4x^3 - 8x^2)} \\
 & \qquad 7x^2 + 3x \\
 & \qquad \underline{-(7x^2 + 14x)} \\
 & \qquad\quad -11x + 4 \\
 & \qquad\quad \underline{-(-11x - 22)} \\
 & \qquad\qquad 26 \leftarrow \text{remainder}
\end{array}
$$

<u>Ans:</u> $\boxed{\dfrac{2x^4 - x^2 + 3x + 4}{x+2} = 2x^3 - 4x^2 + 7x - 11 + \dfrac{26}{x+2}}$

<u>Notice:</u> Only the numbers are important; the variables are only placeholders:

$$
\begin{array}{r l}
 & 2 \quad -4 \quad 7 \quad -11 \\
x+2 \,\big)\!\!\! & 2 \quad 0 \quad -1 \quad 3 \quad 4 \\
 & \underline{-(2 \quad 4)} \\
 & \quad -4 \quad -1 \\
 & \quad \underline{-(-4 \quad -8)} \\
 & \qquad 7 \quad 3 \\
 & \qquad \underline{-(7 \quad 14)} \\
 & \qquad\quad -11 \quad 4 \\
 & \qquad\quad \underline{-(-11 \quad -22)} \\
 & \qquad\qquad 26
\end{array}
$$

shorten work by eliminating duplication and change $x+2$ to "$-2$" so we can add rather then subtract.

## E. Synthetic Division

By stripping away even more, we get a shortcut for division by <u>monic linear divisors</u>.

This is called <u>synthetic division</u>.

Repeat the previous example ...

Divide $2x^4 - x^2 + 3x + 4$ by $x+2$:

$$
\begin{array}{r|rrrrrl}
-2 & 2 & 0 & -1 & 3 & 4 & \leftarrow \text{coefficients} \\
 & \downarrow & -4 & 8 & -14 & 22 & \leftarrow \text{add across this line} \\
\hline
 & 2 & -4 & 7 & -11 & \boxed{26} & \leftarrow \text{remainder}
\end{array}
$$

"x+2" changed to c=-2

We get the same answer with less work.

Reading the answer from right to left, the last entry is the remainder, then the constant, $x, x^2, x^3$ terms

i.e. $$2x^3 - 4x^2 + 7x - 11 + \frac{26}{x+2}$$

<u>Note:</u> This shortcut only works for <u>monic linear divisors</u>. In all other cases, algebraic long division must be used!

## F. Examples

Example 1: Divide $3x^3 - x + 5$ by $x - 1$ using synthetic division

Solution

$$
\begin{array}{r|rrrr}
1 & 3 & 0 & -1 & 5 \\
 &  & 3 & 3 & 2 \\
\hline
 & 3 & 3 & 2 & \boxed{7} \\
 & x^2 & x & c &
\end{array}
$$

Ans: $\boxed{3x^2 + 3x + 2 + \frac{7}{x-1}}$

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Example 2: Divide $2x^4 + 6x^3 + 5x - 3$ by $x + 3$ using synthetic division.

Solution

$$
\begin{array}{r|rrrrr}
-3 & 2 & 6 & 0 & 5 & -3 \\
 &  & -6 & 0 & 0 & -15 \\
\hline
 & 2 & 0 & 0 & 5 & \boxed{-18} \\
 & x^3 & x^2 & x & c &
\end{array}
$$

Ans: $\boxed{2x^3 + 5 - \frac{18}{x+3}}$