

3.3A Review of Dividing Polynomials

A. Dividing Polynomials Using Algebraic Long Division

We use what is called **algebraic long division**.

You do the same steps as in long division of numbers, with a few extra things to consider.

Important Extra Features:

1. Write the polynomials in descending order.
2. If any powers are missing, then **include** zero terms.
These act as placeholders.
3. When subtracting polynomials, put parentheses around the polynomial.
The minus sign affects everything.
4. You stop when the **degree** of the remainder is smaller than the divisor.
5. You need to separate the remainder with an extra + sign (no mixed number!)

This process is easier to do, than to describe.

Look at the following examples . . .

B. Examples

Example 1: Divide $3x^3 + 4x^2 + x + 7$ by $x^2 + 1$

Solution

$$x^2 + 0x + 1 \overline{) 3x^3 + 4x^2 + x + 7} \quad \leftarrow \text{Divide } x^2 \text{ into } 3x^3: 3x$$

$$\begin{array}{r} x^2 + 0x + 1 \overline{) 3x^3 + 4x^2 + x + 7} \\ \underline{3x^3 + 0x^2 + 3x} \\ 3x^3 + 0x^2 + 3x \end{array} \quad \leftarrow \text{Multiply } 3x(x^2 + 0x + 1)$$

Now subtract. Remember parentheses.

$$\begin{array}{r} x^2 + 0x + 1 \overline{) 3x^3 + 4x^2 + x + 7} \\ \underline{-(3x^3 + 0x^2 + 3x)} \\ 4x^2 - 2x + 7 \end{array} \quad \leftarrow \text{Bring down 7}$$

Now divide x^2 into $4x^2$: 4.

$$\begin{array}{r} x^2 + 0x + 1 \overline{) 3x^3 + 4x^2 + x + 7} \\ \underline{-(3x^3 + 0x^2 + 3x)} \\ 4x^2 - 2x + 7 \\ \underline{4x^2 + 0x + 4} \\ -2x + 3 \end{array} \quad \leftarrow \text{Multiply } 4(x^2 + 0x + 1)$$

Now subtract. Remember parentheses.

$$\begin{array}{r} x^2 + 0x + 1 \overline{) 3x^3 + 4x^2 + x + 7} \\ \underline{-(3x^3 + 0x^2 + 3x)} \\ 4x^2 - 2x + 7 \\ \underline{-(4x^2 + 0x + 4)} \\ -2x + 3 \end{array} \quad \leftarrow \text{Stop: smaller degree than } x^2 + 1.$$

Ans $\boxed{3x + 4 + \frac{-2x + 3}{x^2 + 1}}$

Example 2: Divide $8x^3 - x^2 + 4$ by $2x^2 - 3$

Solution

$$2x^2 + 0x - 3 \overline{) 8x^3 - x^2 + 0x + 4} \quad \leftarrow \text{Divide } 2x^2 \text{ into } 8x^3: 4x$$

$$2x^2 + 0x - 3 \overline{) 8x^3 - x^2 + 0x + 4} \\ \quad \quad \quad 4x \\ \quad \quad \quad \underline{8x^3 + 0x^2 - 12x} \quad \leftarrow \text{Multiply } 4x(2x^2 + 0x - 3) \\ \quad \quad \quad \text{Now subtract. Remember parentheses.}$$

$$2x^2 + 0x - 3 \overline{) 8x^3 - x^2 + 0x + 4} \\ \quad \quad \quad 4x \\ \quad \quad \quad \underline{-(8x^3 + 0x^2 - 12x)} \\ \quad \quad \quad \quad \quad -x^2 + 12x + 4 \quad \leftarrow \text{Bring down 4} \\ \quad \quad \quad \quad \quad \text{Now divide } 2x^2 \text{ into } -x^2: -\frac{1}{2}.$$

$$2x^2 + 0x - 3 \overline{) 8x^3 - x^2 + 0x + 4} \\ \quad \quad \quad 4x - \frac{1}{2} \\ \quad \quad \quad \underline{-(8x^3 + 0x^2 - 12x)} \\ \quad \quad \quad \quad \quad -x^2 + 12x + 4 \\ \quad \quad \quad \quad \quad -x^2 + 0x + \frac{3}{2} \quad \leftarrow \text{Multiply } -\frac{1}{2}(2x^2 + 0x - 3) \\ \quad \quad \quad \quad \quad \text{Now subtract. Remember parentheses.}$$

$$2x^2 + 0x - 3 \overline{) 8x^3 - x^2 + 0x + 4} \\ \quad \quad \quad 4x - \frac{1}{2} \\ \quad \quad \quad \underline{-(8x^3 + 0x^2 - 12x)} \\ \quad \quad \quad \quad \quad -x^2 + 12x + 4 \\ \quad \quad \quad \quad \quad -\left(-x^2 + 0x + \frac{3}{2}\right) \\ \quad \quad \quad \quad \quad \underline{\hspace{1.5cm}} \\ \quad \quad \quad \quad \quad 12x + \frac{5}{2} \quad \leftarrow \text{Stop: smaller degree than } 2x^2 - 3.$$

Ans $\boxed{4x - \frac{1}{2} + \frac{12x + \frac{5}{2}}{2x^2 - 3}}$