3.2C Polynomial Functions II

A. Graphing Strategy

With the polynomial factored,

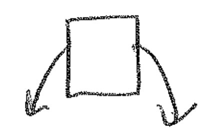
1. Determine the degree, n and the leading coefficient a_n to get the end behavior shape.
2. Each factor $(x-a)^k$ determines an x-intercept $x=a$. Make a table listing the x-intercepts, the multiplicity, and the behavior (i.e. cross/touch and flat/not flat)
3. Find the y-intercept.
4. Mark the data on the graph and connect.

B. Example

Graph f, where $f(x) = -2(x-3)^2(x+1)^3(x+4)$

Solution

1. Degree, $n=6$ and Leading Coefficient, $a_n = -2 < 0$



End Behavior

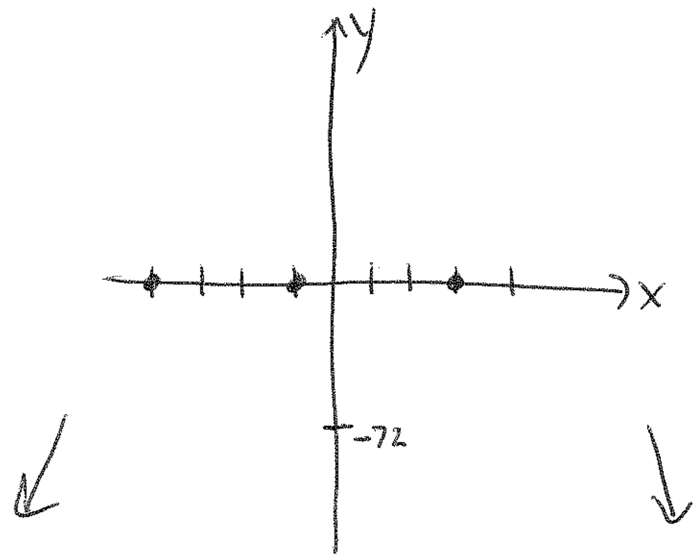
2. Table:

x-intercepts	multiplicity, k	behavior
3	2	touch, flat
−1	3	cross, flat
−4	1	cross, not flat

3. Find y-intercept: set $x=0$:

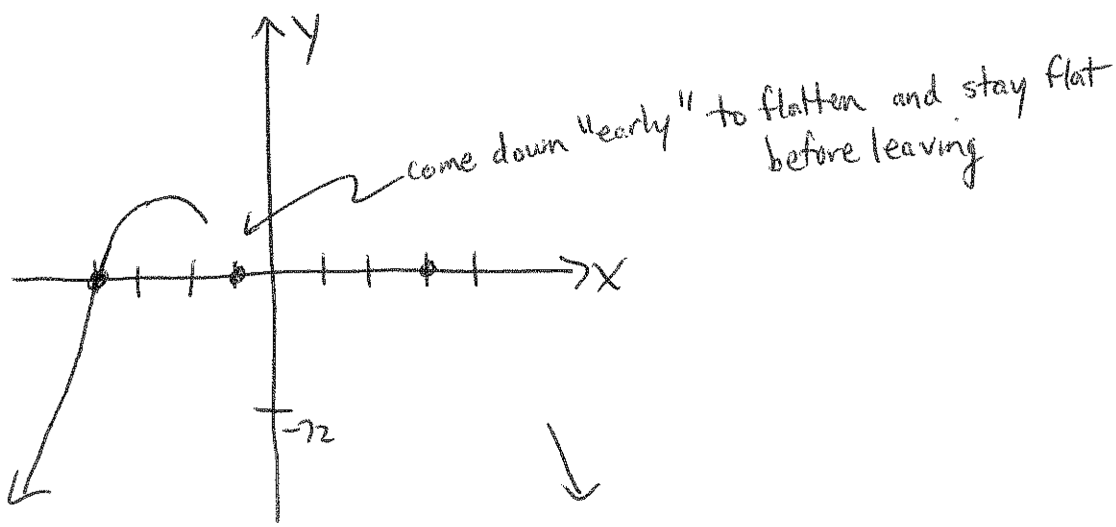
$$f(0) = -2(0-3)^2(0+1)^3(0+4)$$
$$= -2(9)(1)(4)$$
$$= -72$$

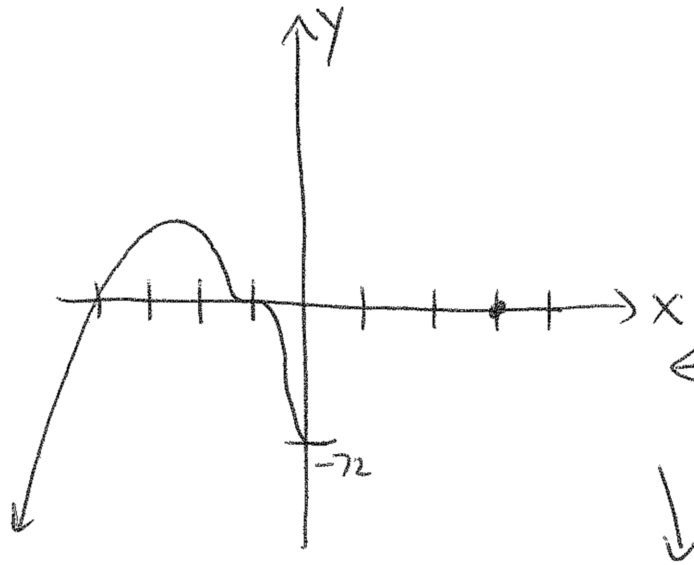
4. Put on graph:



At $x=-4$, cross and not flat: 

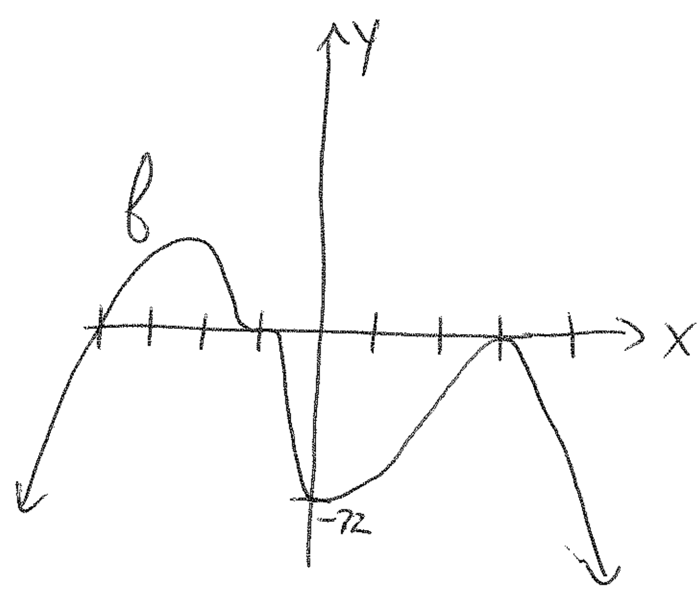
At $x=-1$: cross, flat:





At $x=3$, touch and flat

Ans:



C. Comments And Terminology

1. The relative maxs and mins of the graph (above example has three: two hills, one valley) are called the turning points of the graph.

2. Fact: The number of turning points is at most $n-1$ (In the above example, we have 3, which is ≤ 5)

3. <u>Equivalence of Terminology</u>:

a. <u>x-intercept</u> of the graph of f

b. <u>zero</u> of the function f

c. <u>root</u> of the equation $f(x)=0$

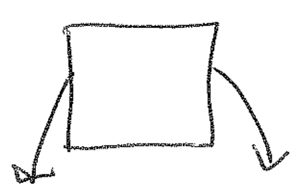
4. For a factor $(x-a)^k$ with $k>1$, "a" is called a <u>repeated zero</u>.

5. If the polynomial is <u>not</u> already factored, it must be factored first. Since this can be difficult for polynomials of high degree and we only have <u>factoring by grouping</u> as a tool for such things, we will study some new <u>advanced factoring methods</u> in the next few sections.

D. Mathematical End Behavior I

Given the overall end behavior for a polynomial function,

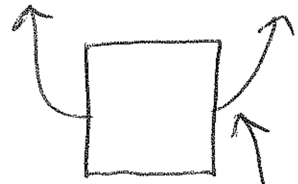
e.g.



<u>falls</u> to the left
<u>falls</u> to the right

we wish to be able to describe this behavior using mathematical language

Consider :



rises to the left
rises to the right

y-values get fantastically large (go to ∞)
as the x-values get fantastically large (go to ∞)

1. Describing the right branch, we say that

$$f(x) \xrightarrow[\text{goes to}]{} \infty \quad \text{as} \quad x \xrightarrow[\text{goes to}]{} \infty$$

2. Similarly, the left branch may be described by

$$f(x) \to \infty \quad \text{as} \quad x \to -\infty$$

E. Mathematical End Behavior II

We first draw the overall shape, then we fill in the following blanks:

$f(x) \to$ ____ as $x \to -\infty$ (left branch)

$f(x) \to$ ____ as $x \to \infty$ (right branch)

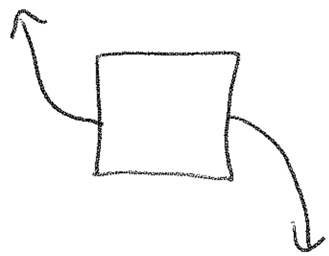
Example: Describe the mathematical end behavior for f where

$$f(x) = -3(x-2)^3(x+1)^2(x-1)(x-3)^5$$

Solution

Degree, $n = 11$

Leading Coefficient, $a_n = -3$



rises to the left
falls to the right

Ans:

$$\boxed{\begin{array}{l} f(x) \to \infty \quad \text{as} \quad x \to -\infty \\ f(x) \to -\infty \quad \text{as} \quad x \to \infty \end{array}}$$