3.2B Polynomial Functions I

A. Definition of a Polynomial Function

f is a polynomial function if

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \quad , \text{where } a_n \neq 0$$

n, the highest power present, is called the degree.

B. Graphs of Polynomial Functions

1. If the degree is ≤ 1, i.e. "no powers", we have the graph of a straight line (discussed in 2.3/2.4)

2. If the degree is 2, we have a quadratic function, and the graph is a parabola (discussed in 3.1)

3. Thus we are mainly now interested in the graphs of polynomial functions where the degree is ≥ 3.

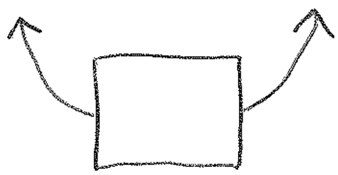
4. Since large values of x (both positively and negatively) are predominantly affected by the highest power present, i.e. the degree, the overall shape (ignoring the x-intercepts on the inside) is the same as that of the power functions.

5. The "overall shape" is called the end behavior.

C. Polynomial End Behavior: Informal Description

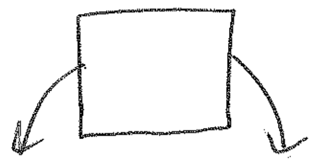
Ignoring translations and the middle of the graph, all polynomial functions have the following appearances:

1. Degree, n, even:



$a_n > 0$

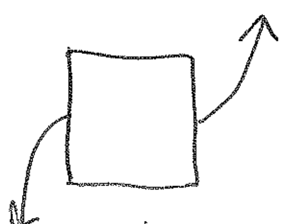
rises to the left
rises to the right



$a_n < 0$
(accounts for x-axis reflection)

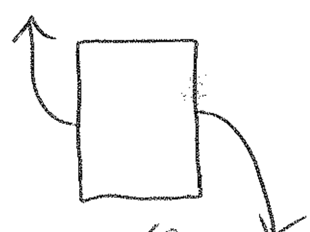
falls to the left
falls to the right

2. Degree, n, odd:



$a_n > 0$

falls to the left
rises to the right



$a_n < 0$
(accounts for x-axis reflection)

rises to the left
falls to the right

It only remains to find out what happens in the middle at the x-intercepts.

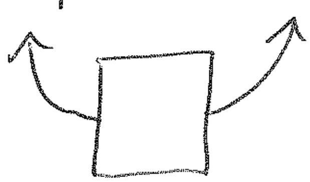
D. Inside Behavior At x-intercepts I

For ease in study, we consider the polynomial $f(x)$ in <u>factored form</u>, i.e.

$$f(x) = 3(x-1)^2(x+2)(x+1)^3$$

<u>Note:</u> The degree is the <u>sum</u> of the factored powers. Thus for the example above, the degree is <u>6</u>.

Since $a_n = a_6 = 3 > 0$ and since $n = 6$, the overall shape is



Moreover, we see that we have x-intercepts of $1, -2$, and -1

E. Inside Behavior At x-intercepts II

The power on the factors is called the <u>multiplicity</u>, symbolized by the letter k.

In the above example:

x-intercepts	multiplicity, k
1	2
-2	1
-1	3

F. Inside Behavior At X-intercepts III

The multiplicity determines "how" the graph hits the x-axis.

1. If k is <u>even</u>, i.e. 2, 4, 6, 8

← graph looks like this at the x-intercept
(i.e. power function with even power)

Thus the graph:

a. <u>touches</u> the x-axis and bounces off

b. is <u>flat</u> (i.e. horizontal) at the x-intercept

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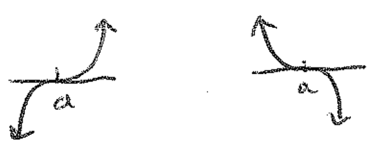
2. If k is <u>odd</u> and <u>greater than 1</u>, i.e. 3, 5, 7, 9



← graph looks like this at the x-intercept
(i.e. power function with odd power ≥ 3)

Thus the graph:

a. <u>crosses</u> the x-axis

b. is <u>flat</u> at the x-intercept

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3. If k=1,

← graph looks like this at the x-intercept
(i.e. power function with power=1)

Thus the graph:

a. <u>crosses</u> the x-axis

b. is not flat at the x-intercepts (just cuts through)

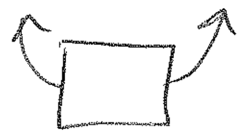
G. Initial Example

Returning to the example

$$f(x) = 3(x-1)^2(x+2)(x+1)^3$$

We know:

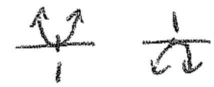
1. Degree, $n=6$

2. End Behavior: 

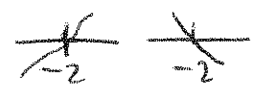
$a_n > 0$

rises to the left
rises to the right

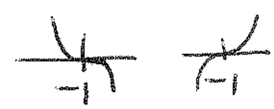
3. x-intercepts:

$x=1$: Since $k=2$, looks like 

touches and flat

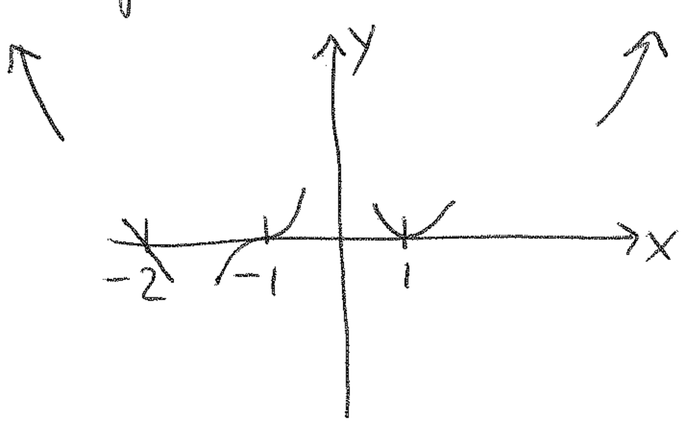
$x=-2$: Since $k=1$, looks like 

crosses, not flat

$x=-1$: Since $k=3$, looks like 

crosses, flat

4. Put together:



← Then connect