3.1A Quadratic Functions I

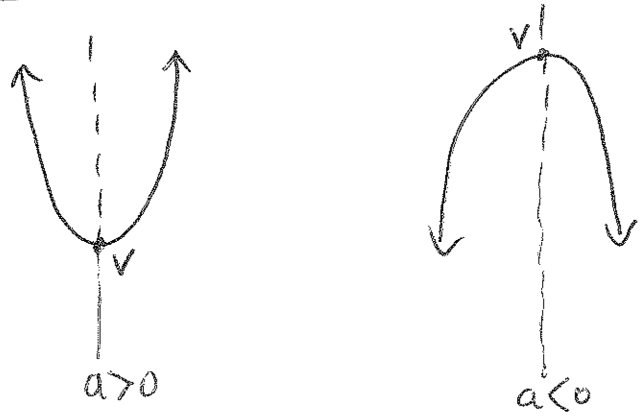
A. Definition of a Quadratic Function

f is a quadratic function if $f(x)=ax^2+bx+c$, $a\neq 0$

The graph is a parabola.

B. Parabolas

1.



2. V is the vertex of the parabola

3. V is the absolute minimum point when $a>0$
   V is the absolute maximum point when $a<0$

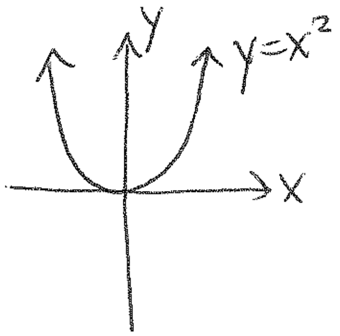
C. Standard Form

1. The standard form for a quadratic function is given by $f(x)=a(x-h)^2+k$, for function f.

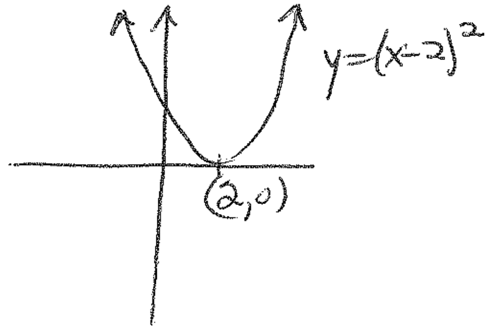
2. Using HSRV, we shift to right h, stretch/shrink by a, flip across x-axis if $a<0$, and shift up by k

Consider the following example...

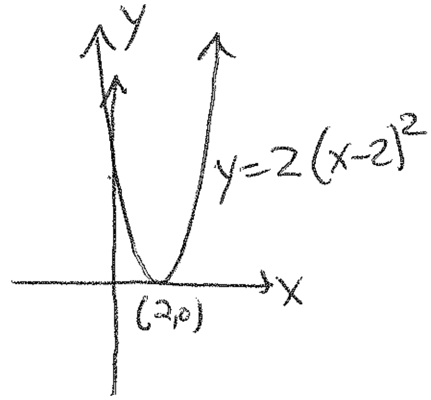
Example: Locate f, where $f(x) = -2(x-2)^2 + 4$



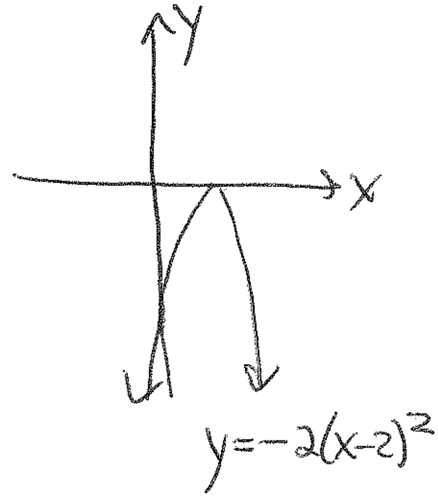
$\xrightarrow{H}$



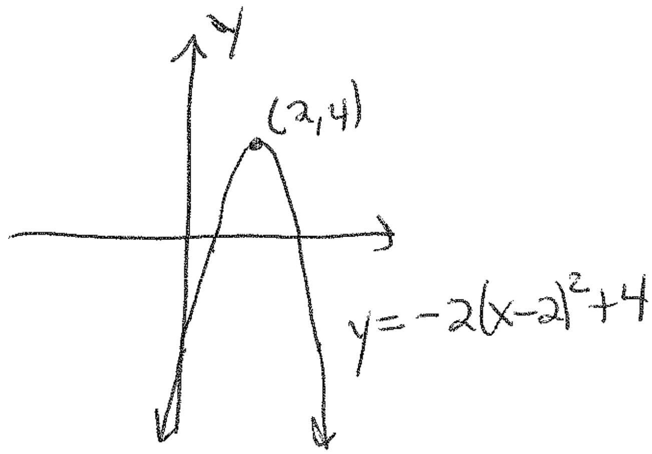
$\xrightarrow{S}$



$\xrightarrow{R}$



$\xrightarrow{V}$



3. <u>Note:</u>

$$\boxed{\begin{array}{l}\text{Vertex is } (h,k) \\ \text{Axis of Symmetry is } x=h\end{array}}$$

D. Method For Graphing Parabolas

1. Locate the vertex of the parabola
   a. Method1: Use completing the square to put quadratic function in standard form
   b. Method 2: Use a formula for the vertex (To come later)

2. Determine the x and y intercepts
3. Plot the known points and connect in a smooth curve, knowing which direction it opens (up/down)

E. Examples

Example1: Graph f, where $f(x) = 2x^2 + 12x + 15$

Solution

1. Locate vertex; put in standard form complete the square method

$$f(x) = 2x^2 + 12x + 15$$
$$f(x) = 2(x^2 + 6x) + 15$$
$$f(x) = 2(x^2 + 6x + 9) + 15 - 18$$
$$f(x) = 2(x+3)^2 - 3$$

vertex: $(-3, -3)$

2. x-intercepts: set $y=0$: $0=2x^2+12x+15$

-or- $0=2(x+3)^2-3$

$$\Rightarrow 2(x+3)^2=3$$

$$(x+3)^2=\frac{3}{2}$$

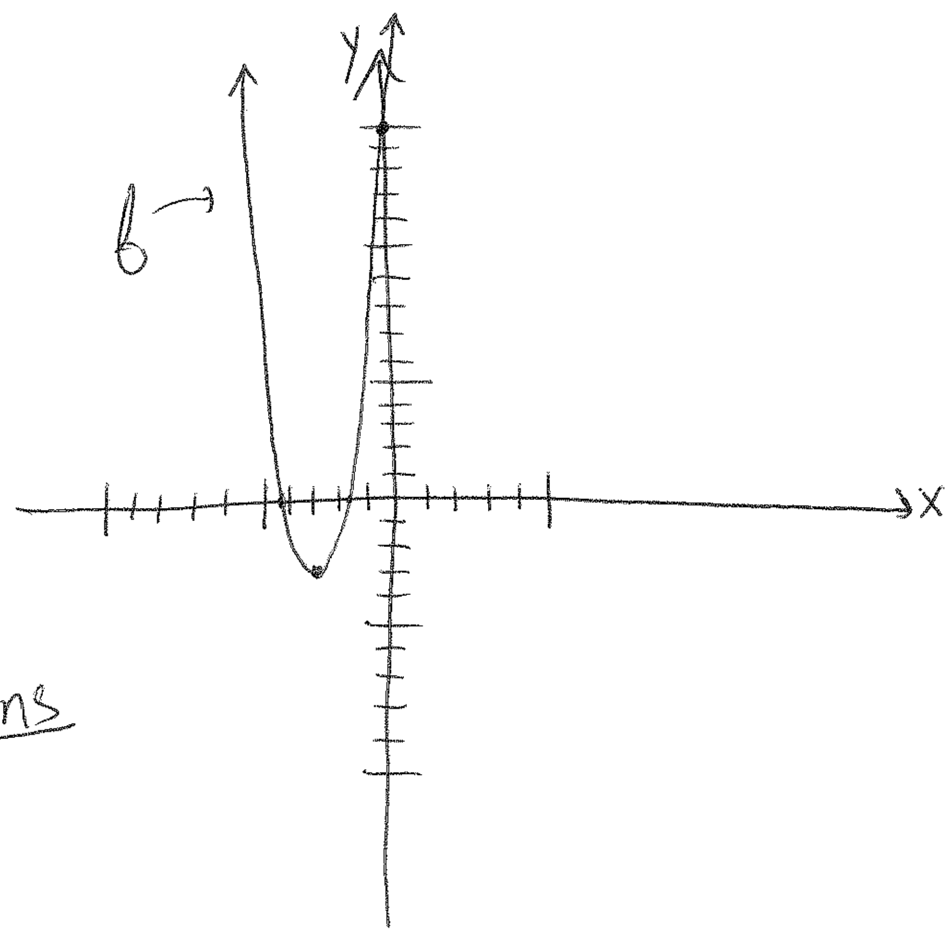
$$x+3=\pm\sqrt{\frac{3}{2}}$$

$$x=-3\pm\sqrt{\frac{3}{2}}=\frac{-6}{2}\pm\frac{\sqrt{3}\cdot\sqrt{2}}{2}$$

$$x=\frac{-6\pm\sqrt{6}}{2}$$

y-intercept: set $x=0$: $f(0)=2(0)^2+12(0)+15=15$

Note: opens upward



Ans

Example 2: Graph f, where $f(x) = -2x^2 + 4x - 7$

Solution

1. Locate vertex; put in standard form
complete the square method

$$f(x) = -2x^2 + 4x - 7$$
$$f(x) = -2(x^2 - 2x) - 7$$
$$f(x) = -2(x^2 - 2x + 1) - 7 + 2$$
$$f(x) = -2(x-1)^2 - 5$$

vertex: $(1, -5)$

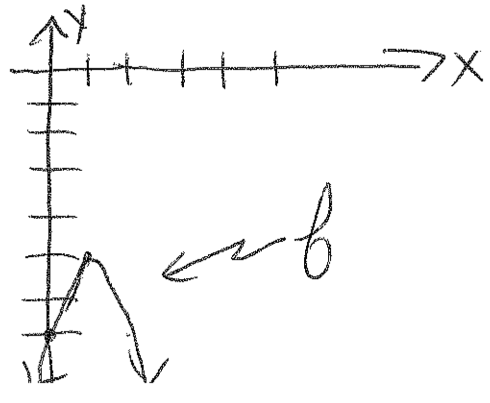
2. x-intercepts: set $y = 0$; $0 = -2x^2 + 4x - 7$
—or— $0 = -2(x-1)^2 - 5$

$$\Rightarrow 2(x-1)^2 = -5$$
$$(x-1)^2 = \frac{-5}{2}$$
$$x - 1 = \pm\sqrt{\frac{-5}{2}}$$ ← complex solutions not on x-axis

no x-intercepts

y-intercepts: set $x = 0$, $f(0) = -2(0)^2 + 4(0) - 7 = -7$

Note: opens downward



Ans

F. Formula For The Vertex

1. Formula for the vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

2. Justification:

$$f(x) = ax^2 + bx + c$$
$$f(x) = a\left(x^2 + \frac{b}{a}x\right) + c$$
$$f(x) = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + c - a\left(\frac{b}{2a}\right)^2$$
$$f(x) = a\left(x + \frac{b}{2a}\right)^2 + \left[c - a\left(\frac{b}{2a}\right)^2\right]$$

vertex: $\left(\frac{-b}{2a}, c - a\left(\frac{b}{2a}\right)^2\right)$

Now $f\left(\frac{-b}{2a}\right) = a\left(\frac{-b}{2a} + \frac{b}{2a}\right)^2 + \left[c - a\left(\frac{b}{2a}\right)^2\right]$

$$= 0 + c - a\left(\frac{b}{2a}\right)^2$$
$$= c - a\left(\frac{b}{2a}\right)^2$$

Thus vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

3. Since the vertex is also (h, k) and the axis of symmetry is $x = h$, we have that the axis of symmetry is $x = \frac{-b}{2a}$