2.7D Inverse Functions II: Reflections

A. Introduction

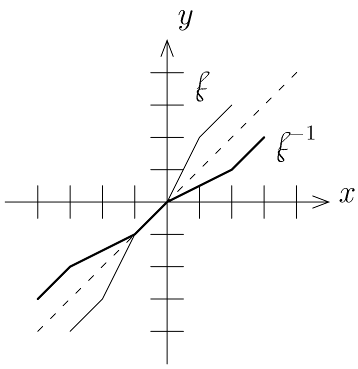
It is sometimes undesirable to examine the inverse function by looking sideways. Thus, for graphical purposes, we can get a "non-sideways" version of the graph of f^{-1} by switching the x and y coordinates. Thus to get a "non-sideways" version of the graph of f^{-1}, we would take each point on the graph of f, say $(3, 2)$ for example and plot $(2, 3)$.

Note: When we want to consider this alternate version of the graph of f^{-1}, we indicate that in our output formula as well. In this case, we switch the letter in the output formula for f^{-1} from y to x; that is, if our original output formula was $f^{-1}(y) = 3y - 2$, our new output formula is $f^{-1}(x) = 3x - 2$.

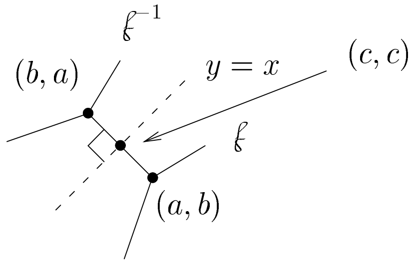
Now let's see what this means geometrically.

B. Graph of the Inverse Function

To get the graph of f^{-1}, we switch the coordinates of each point on the graph of f. Geometrically, this corresponds to **reflecting the graph about the line** $y = x$.



C. Justification of Geometric Interpretation of the Inverse



1. If (a, b) is on f, then (b, a) is on f^{-1}.

2. Connecting these two points, we cut the line $y = x$ at some point (c, c).

3. The slope of the connecting segment is $\frac{a-b}{b-a} = \frac{a-b}{-(a-b)} = -1$.

4. Since the slope of the line $y = x$ is 1, we see that the connecting segment is perpendicular to the line $y = x$.

5. To show that f^{-1} is a mirror image across $y = x$, we just need to show that (a, b) is the same distance from (c, c) as the point (b, a) is . . .

6. Distance from (a, b) to (c, c): $\sqrt{(c-a)^2 + (c-b)^2}$

7. Distance from (b, a) to (c, c): $\sqrt{(c-b)^2 + (c-a)^2} = \sqrt{(c-a)^2 + (c-b)^2}$

Thus (b, a) is the mirror image of (a, b) across the line $y = x$, so the graph of f^{-1} is the reflection of the graph of f across the line $y = x$.