2.7A Inverses

A. Definition of Inverses

Two functions f and g are called **inverses** if two conditions are met:

1. $(f \circ g)(x) = x$

2. $(g \circ f)(x) = x$

Thus f and g undo each other!

B. Examples

Example 1: Are f and g inverses, where $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x-1}$?

Solution

Check the two conditions!

1. $(f \circ g)(x) = f(g(x)) = f(\sqrt[3]{x-1}) = (\sqrt[3]{x-1})^3 + 1 = x - 1 + 1 = x$

2. $(g \circ f)(x) = g(f(x)) = g(x^3 + 1) = \sqrt[3]{(x^3+1)-1} = \sqrt[3]{x^3} = x$

Ans $\boxed{\text{YES, } f \text{ and } g \text{ are inverses}}$

Example 2: Are f and g inverses, where $f(x) = x^2$ and $g(x) = \sqrt{x}$?

Solution

Check the two conditions!

1. $(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$

2. $(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$

Both conditions are not met, so . . .

Ans $\boxed{f \text{ and } g \text{ are NOT inverses}}$
