2.6 D Domain of Composition: From Output Formulas

A. Introduction

In this section, we will discuss how to find dom$(f \circ g)$ from output formulas.

Unfortunately, finding dom$(f \circ g)$ from graphs requires a different recipe (see next section)

B. Motivation

Since $(f \circ g)(x) = f(g(x))$, the domain must consist of

1. values that g accepts [dom g]
2. also, $g(x)$ must be accepted by f; this occurs when $f(g(x))$ is defined (i.e. dom$(f \circ g)_{\text{incorrect}}$ ← found by looking at the output formula)

C. Method for Finding dom$(f \circ g)$

1. First find dom(g) [this is the function on the right]
2. Find $(f \circ g)(x)$ and use this to find dom$(f \circ g)_{\text{incorrect}}$
3. Intersect dom g and dom$(f \circ g)_{\text{incorrect}}$ with AND

Note: dom f [the left function] does not get used!

D. Examples

Example 1: Let $f(x) = \dfrac{1}{x^2 - 1}$ and $g(x) = \sqrt{x+3}$.

Find a) $(f \circ g)(x)$

b) $\mathrm{dom}(f \circ g)$

Solution

a) $(f \circ g)(x) = f(g(x)) = f(\sqrt{x+3}) = \dfrac{1}{(\sqrt{x+3})^2 - 1}$

$$= \frac{1}{x+3-1}$$

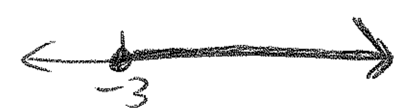
$$\boxed{(f \circ g)(x) = \frac{1}{x+2}}$$

b) Find $\mathrm{dom}(f \circ g)$:

1. First find dom g (the right function)

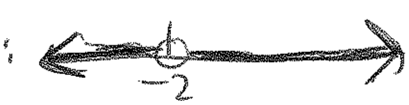
A. Den ✓

B. Even Root: set $x + 3 \geq 0 \Rightarrow x \geq -3$

dom g: 

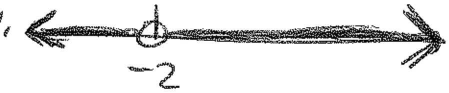
2. Find dom(f∘g) incorrect:

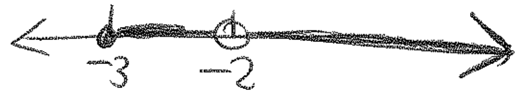
Since $(f \circ g)(x) = \dfrac{1}{x+2}$, $x \neq -2$!

dom(f∘g) incorrect: 

3. Intersect with AND:

dom g: 

dom(fog) incorrect: 

AND 

$$\boxed{\text{dom}(f \circ g) = [-3, -2) \cup (-2, \infty)}$$

<u>Note:</u> dom f is not used.

<u>Example 2:</u> Let $f(x) = \dfrac{x+3}{x-1}$ and $g(x) = \dfrac{2x+1}{x+3}$.

Find a) $(f \circ g)(x)$

b) dom$(f \circ g)$

<u>Solution</u>

a) $$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+1}{x+3}\right)$$

$$= \frac{\left(\frac{2x+1}{x+3}\right) + 3}{\left(\frac{2x+1}{x+3}\right) - 1} = \frac{\frac{2x+1}{x+3} + \frac{3(x+3)}{x+3}}{\frac{2x+1}{x+3} - \frac{x+3}{x+3}}$$

$$= \frac{\frac{5x+10}{x+3}}{\frac{x-2}{x+3}} = \frac{5x+10}{x+3} \cdot \frac{x+3}{x-2} = \boxed{\frac{5(x+2)}{x-2}}$$

b) Find dom$(f \circ g)$:

1. First find dom g [the right function]

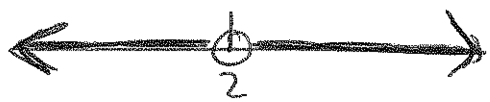
   A. Den: Throw away $x = -3$

   B. Even Root: ✓

   dom g: 

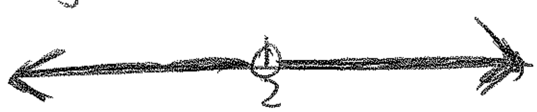
2. Find dom$(f \circ g)$incorrect:

   Since $(f \circ g)(x) = \dfrac{5(x+2)}{x-2}$, $x \neq 2$!

   dom$(f \circ g)$incorrect: 

3. Intersect with AND:

   dom g: 

   dom$(f \circ g)$incorrect: 

   AND 

$$\boxed{\text{dom}(f \circ g) = (-\infty, -3) \cup (-3, 2) \cup (2, \infty)}$$

<u>Note:</u> dom f is never used

Example 3: Let $f(x)=\sqrt{x^2+8}$ and $g(x)=\sqrt{x^2-9}$.

Find
a) $(f\circ g)(x)$
b) $(g\circ f)(x)$
c) $\text{dom}(f\circ g)$
d) $\text{dom}(g\circ f)$

Solution

a) $(f\circ g)(x)=f(g(x))=f\left(\sqrt{x^2-9}\right)=\sqrt{\left(\sqrt{x^2-9}\right)^2+8}=\sqrt{x^2-9+8}=\boxed{\sqrt{x^2-1}}$

b) $(g\circ f)(x)=g(f(x))=g\left(\sqrt{x^2+8}\right)=\sqrt{\left(\sqrt{x^2+8}\right)^2-9}=\sqrt{x^2+8-9}=\boxed{\sqrt{x^2-1}}$

Note: In this case, $(f\circ g)(x)=(g\circ f)(x)$ [this doesn't always happen!]

Even so, we will still see that $f\circ g\neq g\circ f$ because the domains will be different.

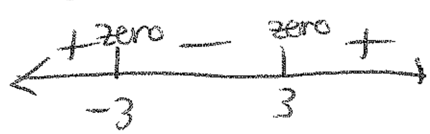
c) $\text{dom}(f\circ g)$:

1. First find dom g [the right function]:

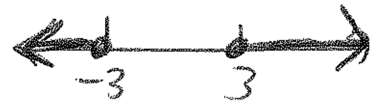
A. Den ✓
B. Even Root: set inside ≥ 0

$x^2-9\geq 0$ ← Quadratic Inequality!

$(x+3)(x-3)\geq 0$ Breakpoints: $-3, 3$



So $\text{dom } g = (-\infty, -3] \cup [3, \infty)$



2. Find $\text{dom}(f \circ g)_{\text{incorrect}}$:

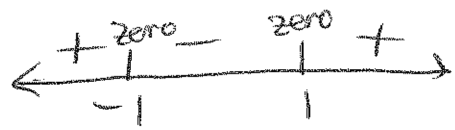
Since $(f \circ g)(x) = \sqrt{x^2 - 1}$:

A. Den ✓

B. Even Root: set inside ≥ 0

$x^2 - 1 \geq 0$ ⟸ Quadratic Inequality!

$(x+1)(x-1) \geq 0$ Breakpoints: $-1, 1$



So $\text{dom}(f \circ g)_{\text{incorrect}} = (-\infty, -1] \cup [1, \infty)$

3. Intersect with AND

dom g

$\text{dom}(f \circ g)_{\text{incorrect}}$

AND

$$\boxed{\text{dom}(f \circ g) = (-\infty, -3] \cup [3, \infty)}$$

d) $\text{dom}(g \circ f)$:

1. First find dom f [the right function]

A. Den ✓

B. Even Root: set inside ≥ 0

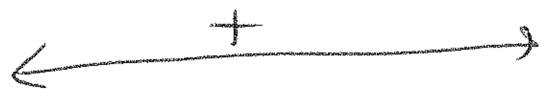
$x^2+8 \geq 0 \Leftarrow$ Quadratic Inequality

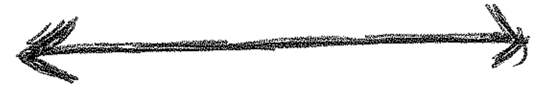
x^2+8 is prime, so

$x^2+8=0 \Rightarrow x^2=-8 \Rightarrow x=\pm\sqrt{-8}=\pm i\sqrt{8}=\pm 2i\sqrt{2}$

no breakpoints

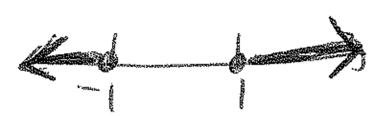
Test whole line: 



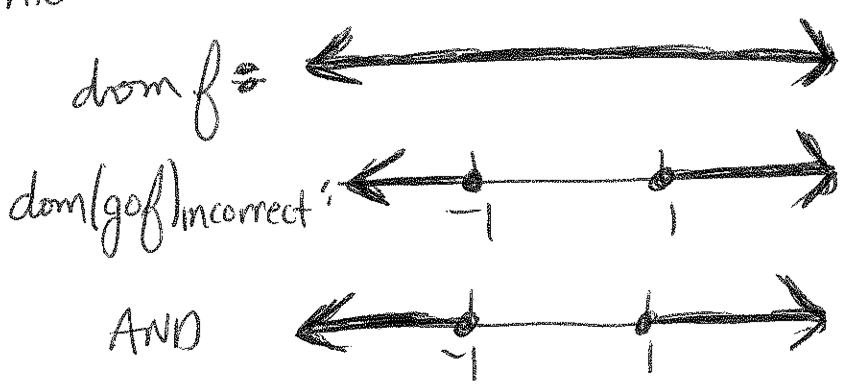
so $\operatorname{dom} f = (-\infty,\infty)$ 

2. Find $\operatorname{dom}(g\circ f)_{\text{incorrect}}$:

Since $(g\circ f)(x)=\sqrt{x^2-1}$, and we solved $x^2-1\geq 0$ already in part (c)

$\operatorname{dom}(g\circ f)_{\text{incorrect}} = (-\infty,-1]\cup[1,\infty)$ 

3. Intersect with AND



$\boxed{\operatorname{dom}(g\circ f) = (-\infty,-1]\cup[1,\infty)}$