

2.6C Function Composition

A. Introduction

We consider a new function combination that we define called function composition, given the symbol " $\circ$ "

B. Defining Relationships

1. We define the function $f \circ g$ (called f composed with g) to be the function with y -value $(f \circ g)(x)$ given by

$$(f \circ g)(x) = f(g(x))$$

2. We define the function $g \circ f$ (called g composed with f) to be function with y -value $(g \circ f)(x)$ given by

$$(g \circ f)(x) = g(f(x))$$

Note: Don't get attached to the letters!

For instance, $(k \circ h)(x) = k(h(x))$

C. Examples

Example: Find $(f \circ g)(x)$ and $(g \circ f)(x)$ where $f(x) = 2x + 1$ and $g(x) = 3x - 2$

Solution

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) = f(3x - 2) = 2(3x - 2) + 1 \\ &= 6x - 4 + 1 = \boxed{6x - 3}\end{aligned}$$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) = g(2x+1) = 3(2x+1) - 2 \\ &= 6x + 3 - 2 \\ &= \boxed{6x-1}\end{aligned}$$

Note: $(f \circ g)(x) = 6x - 3$ but $(g \circ f)(x) = 6x - 1$
order matters!

Example 2: Find $(f \circ g)(x)$ and $(g \circ f)(x)$ where $f(x) = \sqrt{x+1}$ and $g(x) = \frac{2}{x}$

Solution

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) = f\left(\frac{2}{x}\right) = \sqrt{\frac{2}{x} + 1} \\ &= \sqrt{\frac{2}{x} + \frac{x}{x}} = \sqrt{\frac{2+x}{x}} \\ &= \frac{\sqrt{2+x}}{\sqrt{x}} = \frac{\sqrt{2+x}}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} \\ &= \boxed{\frac{\sqrt{x(2+x)}}{x}}\end{aligned}$$

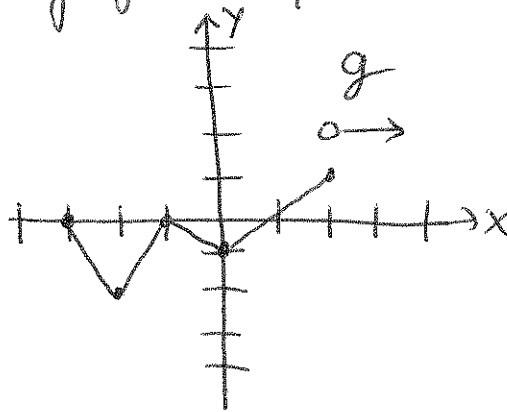
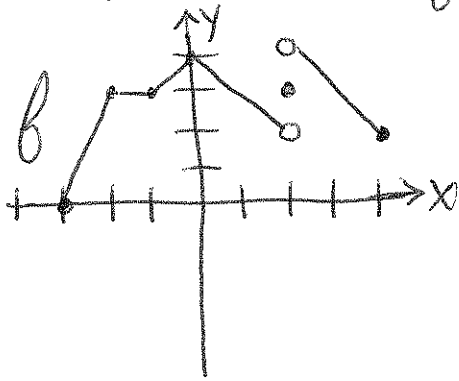
$$\begin{aligned}(g \circ f)(x) &= g(f(x)) = g(\sqrt{x+1}) = \frac{2}{\sqrt{x+1}} \\ &= \frac{2}{\sqrt{x+1}} \cdot \frac{\sqrt{x+1}}{\sqrt{x+1}} \\ &= \boxed{\frac{2\sqrt{x+1}}{x+1}}\end{aligned}$$

Example 3: Let $f(x) = 3 + x^2$ and $g(x) = x - 1$
Find $(g \circ f)(0)$

Solution

$$(g \circ f)(0) = g(\underbrace{f(0)}_{\substack{\text{evaluate } x=0 \\ \text{in } f(x) \text{ output} \\ \text{formula}}}) = g(3) = 3 - 1 = \boxed{2}$$

Example 4: Consider f and g given by the following graphs:



Then find
a) $(f \circ g)(2)$
b) $(g \circ f)(-3)$

Solution

$$\text{a) } (f \circ g)(2) = f(\underbrace{g(2)}_{\substack{\text{y-value on } g \\ \text{when } x=2}}) = f(\underbrace{1}_{\substack{\text{y-value on } f \\ \text{when } x=1}}) = \boxed{3}$$

$$\text{b) } (g \circ f)(-3) = g(\underbrace{f(-3)}_{\substack{\text{y-value on } f \\ \text{when } x=-3}}) = g(\underbrace{0}_{\substack{\text{y-value on } g \\ \text{when } x=0}}) = \boxed{-1}$$