2.6B Domain of Combined Functions

A. Introduction

1. The domains of $f+g$, $f-g$, fg, $\frac{f}{g}$, etc. can <u>not</u> be found by just looking at the output formulas!

2. <u>Reason</u>: Consider $f(x) = x - \sqrt{x}$ and $g(x) = \sqrt{x}$

   Then $(f+g)(x) = f(x) + g(x) = (x - \sqrt{x}) + \sqrt{x} = x$

   With no denominators or even roots for $(f+g)(x) = x$, you might <u>think</u> the domain is $(-\infty, \infty)$.

   This is wrong!

   <u>Why</u>?

   <u>Note</u>: $(f+g)(-2) = \underset{\text{undefined!}}{f(-2)} + \underset{\text{undefined!}}{g(-2)}$

B. Domain of $f+g$, $f-g$, and fg

DO <u>NOT</u> LOOK AT THE FINAL OUTPUT FORMULA!

Instead:

1. Find dom f and dom g SEPARATELY.
2. Intersect the two domains in AND.

<u>Note</u>: By virtue of the recipe, dom$(f+g)$, dom$(f-g)$, and dom(fg) are <u>always</u> <u>the same answer</u>!

C. Example

Let $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{1-x} - \frac{1}{x}$

Find a) $(f+g)(x)$

b) $\text{dom}(f+g)$

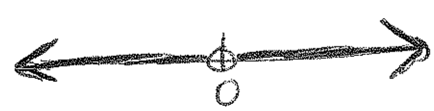
<u>Solution</u>

a) $(f+g)(x) = f(x) + g(x) = \left(\frac{1}{x}\right) + \left(\frac{1}{1-x} - \frac{1}{x}\right) = \boxed{\frac{1}{1-x}}$

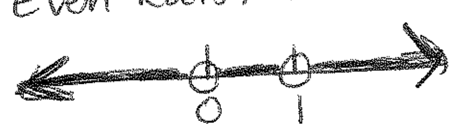
b) Find $\text{dom}(f+g)$:

1. Find dom f and dom g separately:

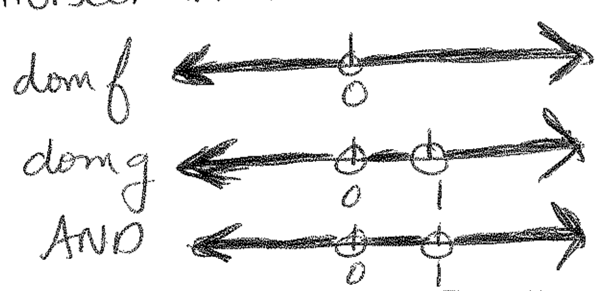
dom f: 1. Den: $x \neq 0$
2. Even Roots: none



dom g: 1. Den: $x \neq 1, 0$
2. Even Roots: none



2. Intersect in AND:



$\boxed{\text{dom}(f+g) = (-\infty, 0) \cup (0, 1) \cup (1, \infty)}$ ⇐ different than output formula from part (a)!

D. Domain of $\frac{f}{g}$

DO <u>NOT</u> LOOK AT THE FINAL OUTPUT FORMULA!

Instead:

1. Find dom f and dom g SEPARATELY.
2. Intersect the two domains in AND.
3. From the answer, additionally throw away any x-values that make $g(x)=0$. (i.e. set $g(x)=0$ and throw away)

E. Example

Let $f(x)=\dfrac{x+1}{\sqrt{x+3}}$ and $g(x)=\dfrac{x-2}{\sqrt{x+3}}$

Find

a) $\left(\frac{f}{g}\right)(x)$

b) dom$\left(\frac{f}{g}\right)$

<u>Solution</u>

a) $$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{\frac{x+1}{\sqrt{x+3}}}{\frac{x-2}{\sqrt{x+3}}}=\frac{x+1}{\sqrt{x+3}}\cdot\frac{\sqrt{x+3}}{x-2}=\boxed{\frac{x+1}{x-2}}$$

b) Find dom$\left(\frac{f}{g}\right)$:

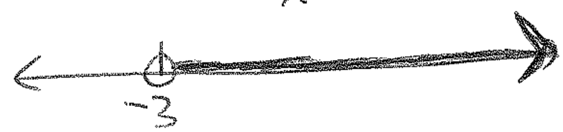
Again, don't look at the answer in part (a)!

1. Find dom f and dom g separately:

dom f: 1. Den: $x+3=0 \Rightarrow$ Throw away $x=-3$

2. Even Root: set inside ≥ 0

$$x+3 \geq 0$$
$$x \geq -3$$

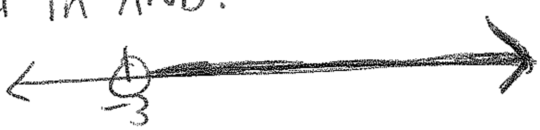


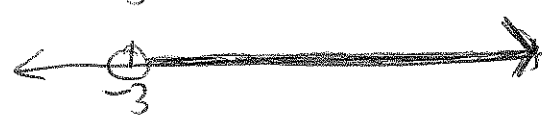
dom g: 1. Den: $x+3=0 \Rightarrow$ Throw away $x=-3$

2. Even Root: set inside ≥ 0

$$x+3 \geq 0$$
$$x \geq -3$$

2. Intersect in AND:

dom f 

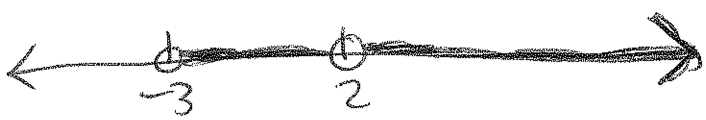
dom g 

AND 

3. Throw away x values where $g(x)=0$:

$$g(x)=0 \Rightarrow \frac{x-2}{\sqrt{x+3}}=0 \Rightarrow \sqrt{x+3}\left(\frac{x-2}{\sqrt{x+3}}\right)=\sqrt{x+3}\,(0)$$

$$\Rightarrow x-2=0 \Rightarrow x=2 \quad \text{Throw away}$$



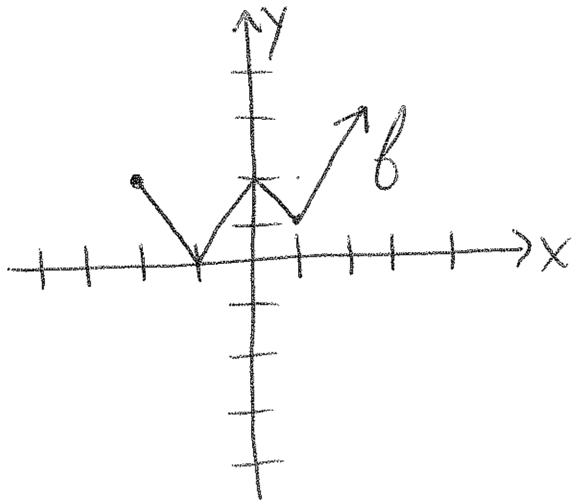
Ans: $\boxed{\text{dom}\left(\frac{f}{g}\right)=(-3,2)\cup(2,\infty)}$ ← different than output formula from part (a)!

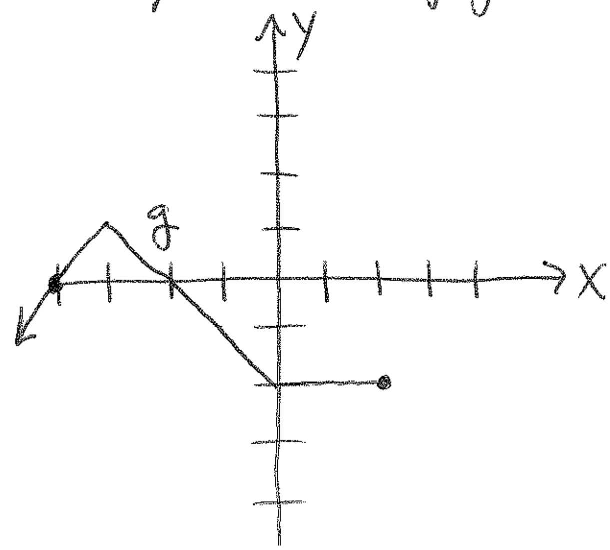
F. Examples Using Graphs

We follow the same recipes, but we need to use the graphs to get our information.

G. Examples

Example 1: Let f and g be given by the following graphs





Find a) $\text{dom}(f+g)$

b) $\text{dom}(f-g)$

c) $\text{dom}(fg)$

d) $\text{dom}\left(\frac{f}{g}\right)$

e) $\text{dom}\left(\frac{g}{f}\right)$

Solution

a) 1. Find dom f and dom g separately

By looking at the graphs,

dom $f = [-2, \infty)$

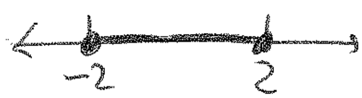
dom $g = (-\infty, 2]$

2. Intersect in AND

dom f

dom g

AND



$$\boxed{\text{dom}(f+g) = [-2, 2]}$$

b) Since dom$(f-g)$ is found by the same recipe,

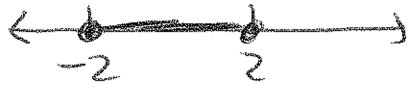
$$\boxed{\text{dom}(f-g) = [-2, 2]}$$

c) Since dom(fg) is found by the same recipe,

$$\text{dom}(fg) = [-2, 2]$$

d) dom$\left(\frac{f}{g}\right)$: First two steps are the same recipe, so we get

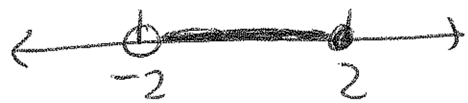
Steps 1 & 2:



Step 3. Throw away where $g(x)=0$

[observe where g hits the x-axis]

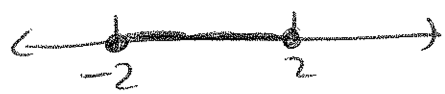
Throw away $x=-4,-2$:



$$\boxed{\mathrm{dom}\left(\frac{f}{g}\right)=(-2,2]}$$

e) $\mathrm{dom}\left(\frac{g}{f}\right)$: First two steps are the same recipe, so we get

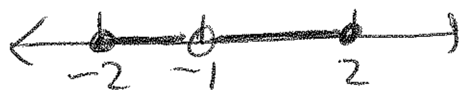
Steps 1 & 2:



Step 3: Throw away where $f(x)=0$ [denominator!]

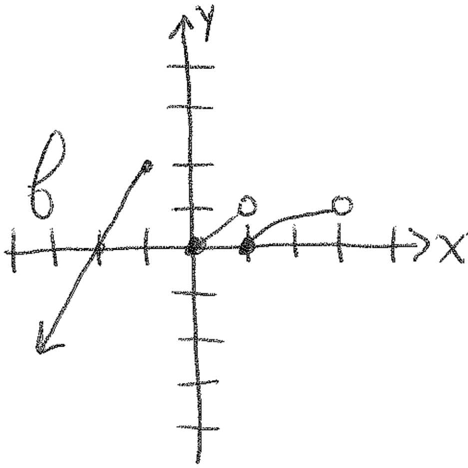
[observe where f hits the x-axis]

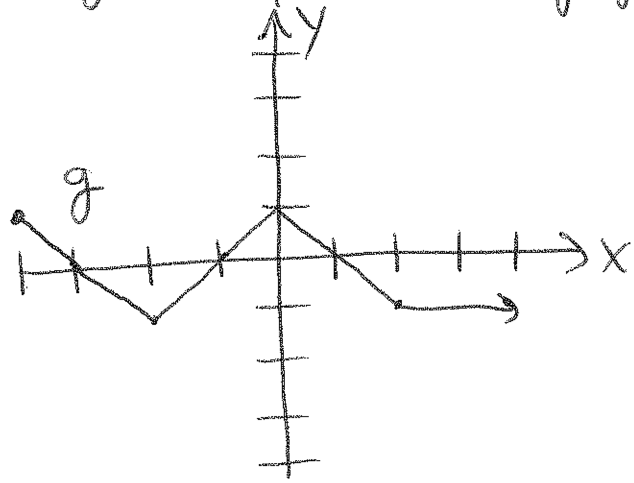
Throw away $x=-1$:



$$\boxed{\mathrm{dom}\left(\frac{g}{f}\right)=[-2,-1)\cup(-1,2]}$$

Example 2: Let f and g be given by the following graphs



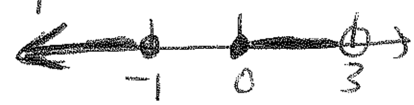


Find $\mathrm{dom}\left(\frac{f}{g}\right)$

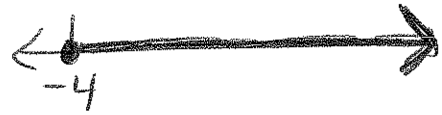
Solution

1. Find $\mathrm{dom}\, f$ and $\mathrm{dom}\, g$ separately:

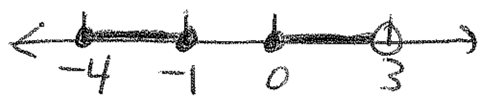
   $\mathrm{dom}\, f$: $(-\infty, -1] \cup [0, 3)$



   $\mathrm{dom}\, g$: $[-4, \infty)$

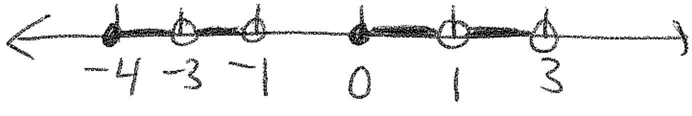


2. Intersect in AND:



3. Now throw away where $g(x) = 0$
   (where g hits the x-axis)

   Throw away $x = -3, -1, 1$



Ans: $\boxed{\mathrm{dom}\left(\frac{f}{g}\right) = [-4, -3) \cup (-3, -1) \cup [0, 1) \cup (1, 3)}$