2.6A Combining Functions

A. Introduction

We have no rules for how to combine functions.
Thus, we need to "invent" rules.
The rules we create are based on our intuition of what we'd like their y-values to be.

B. Defining Relationships

1. We define the function $f+g$ to be the function with y-value $(f+g)(x)$ given by
$$(f+g)(x) = f(x) + g(x)$$
2. We define the function $f-g$ to be the function with y-value $(f-g)(x)$ given by
$$(f-g)(x) = f(x) - g(x)$$
3. We define the function fg (or $f \cdot g$) to be the function with y-value $(fg)(x)$ given by
$$(fg)(x) = f(x) \cdot g(x)$$
4. We define the function $\frac{f}{g}$ to be function with y-value $\left(\frac{f}{g}\right)(x)$ given by
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Note: These are new rules we invented.

C. Examples

Example 1: Find $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$, $\left(\frac{f}{g}\right)(x)$ where

$$f(x)=2x+1 \quad \text{and} \quad g(x)=3x-2$$

Solution

$$(f+g)(x)=f(x)+g(x)=(2x+1)+(3x-2)=\boxed{5x-1}$$

$$(f-g)(x)=f(x)-g(x)=(2x+1)-(3x-2)=\boxed{-x+3}$$

$$(fg)(x)=f(x)\cdot g(x)=\boxed{(2x+1)(3x-2)} \quad \text{(or expand it)}$$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\boxed{\frac{2x+1}{3x-2}}$$

Example 2: Find $(k+h)(x)$, $(k-h)(x)$, $(kh)(x)$, $\left(\frac{k}{h}\right)(x)$ where

$$k(x)=\sqrt{x+1} \quad \text{and} \quad h(x)=\frac{2}{x}$$

Solution

Not being attached to the letters in the definitions, we use the exact same pattern!

$$(k+h)(x)=k(x)+h(x)=\boxed{\sqrt{x+1}+\frac{2}{x}}$$

$$(k-h)(x)=k(x)-h(x)=\boxed{\sqrt{x+1}-\frac{2}{x}}$$

$$(kh)(x)=k(x)\cdot h(x)=\left(\sqrt{x+1}\right)\left(\frac{2}{x}\right)=\boxed{\frac{2\sqrt{x+1}}{x}}$$

$$\left(\frac{k}{h}\right)(x)=\frac{k(x)}{h(x)}=\frac{\sqrt{x+1}}{\frac{2}{x}}=\left(\sqrt{x+1}\right)\cdot\left(\frac{x}{2}\right)=\boxed{\frac{x\sqrt{x+1}}{2}}$$

D. Evaluating At Specific Values

If you want to find the output of one of these new functions at only a few specific values, it is often easier to use the definitions directly, rather than finding the output formula first.

E. Example

Given $f(x) = 3 + x^2$ and $g(x) = x - 1$

Find a) $(fg)(-1)$

b) $\left(\frac{g}{f}\right)(0)$

Solution

a) $(fg)(-1) = f(-1) \cdot g(-1) = (3 + (-1)^2) \cdot (-1 - 1)$

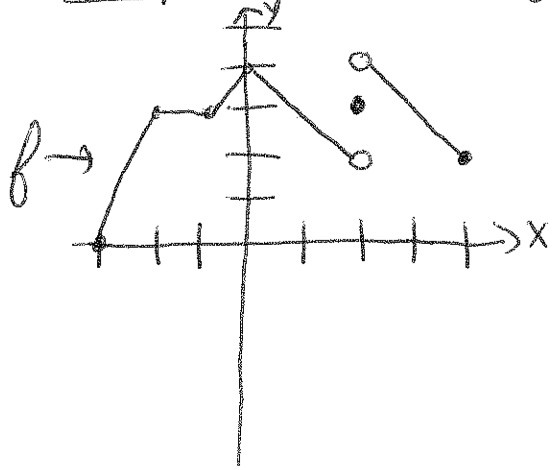
$= (3 + 1)(-2) = \boxed{-8}$

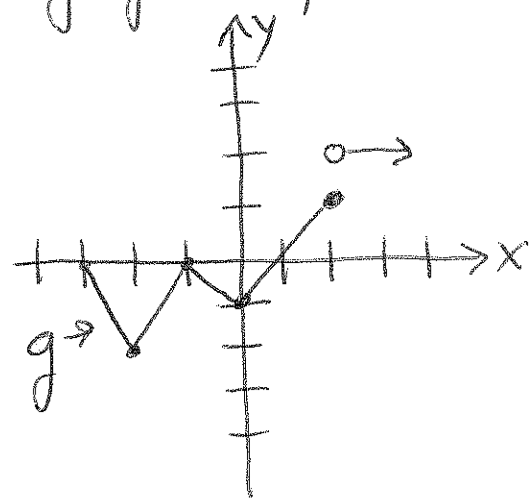
b) $\left(\frac{g}{f}\right)(0) = \frac{g(0)}{f(0)} = \frac{0 - 1}{3 + 0^2} = \boxed{\frac{-1}{3}}$

F. Examples Using Graphs

We can find the outputs to these new functions at specific points, even if we are given just the graphs of f and g.
We, again, use the defining relationships.

<u>Example:</u> Consider f and g given by the following graphs:





Then find a) $(f-g)(2)$

b) $\left(\frac{g}{f}\right)(0)$

<u>Solution</u>

a) $(f-g)(2) = f(2) - g(2) = 3 - 1 = \boxed{2}$

y-value on f for $x=2$

y-value on g for $x=2$

b) $\left(\frac{g}{f}\right)(0) = \frac{g(0)}{f(0)} = \frac{-1}{4} = \boxed{\frac{-1}{4}}$

y-value on g when $x=0$

y-value on f when $x=0$