2.5F HSRV Transformation of Domain and Range

4. Discussion

Suppose you have a transformed function that you want the domain and range of ...

We have the following methods:

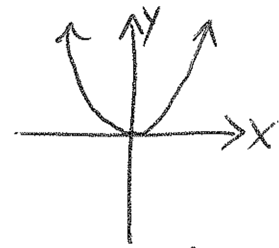
1. If you have the output formula $f(x)$, you can use the techniques discussed previously in 2.1/2.2H and 2.1/2.2I
2. Obtain the graph of the transformed function (2.5C and 2.5D) Then read the domain and range off of the graph, as in Section 2.1/2.2N.
3. <u>Third Method (To be discussed in this section)</u>:
   A. obtain the domain and range of the base graph, either using 2.1/2.2H and I or 2.1/2.2N.
   B. Perform HSRV transformations to the domain and range (as sets of x and y coordinates)
   C. The final "transformed" domain and range is the desired result.

B. Examples

Example 1: Find dom f and rng f if $f(x) = 2(x-3)^2 + 4$ by using HSRV transformations.

Solution

Base Graph: $y = x^2$



dom $f_b = (-\infty, \infty)$

rng $f_b = [0, \infty)$

H: $x^2 \longmapsto (x-3)^2$

move graph right 3, x-values +3
y-values fixed

dom $f_h =$ "$(-\infty+3, \infty+3)$" $= (-\infty, \infty)$

rng $f_h = [0, \infty)$

S: $(x-3)^2 \longmapsto 2(x-3)^2$

stretch vertically by factor of 2; x-values fixed
y-values times 2

dom $f_s = (-\infty, \infty)$

rng $f_s =$ "$[0\cdot 2, \infty\cdot 2]$" $= [0, \infty)$

R: none

V: $2(x-3)^2 \longmapsto 2(x-3)^2+4$

up 4, x-values fixed
y-values up 4

$\text{dom } f_V = (-\infty, \infty)$

$\text{rng } f_V = \text{"}[0+4, \infty+4)\text{"}$
$= [4, \infty)$

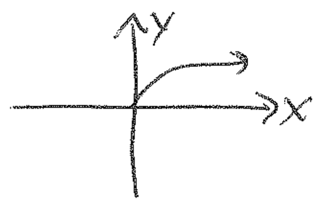
<u>Ans</u>: $\boxed{\begin{array}{l}\text{dom } f = (-\infty, \infty) \\ \text{rng } f = [4, \infty)\end{array}}$

⟸ <u>Note</u>: same result as by 2.1/2.2 H and I

<u>Example 2</u>: Find dom f and rng f if $f(x) = 2 - \frac{1}{3}\sqrt{4 - \frac{x}{2}}$ by using HSRV transformations.

<u>Solution</u>

Base Graph: $y = \sqrt{x}$



$\text{dom } f_b = [0, \infty)$

$\text{rng } f_b = [0, \infty)$

H: $\sqrt{x} \longmapsto \sqrt{x+4} = \sqrt{4+x}$

move graph left 4, x-values −4
y-values fixed

$\text{dom } f_h = \text{"}[0-4, \infty-4)\text{"}$
$= [-4, \infty)$

$\text{rng } f_h = [0, \infty)$

S1: $\sqrt{4+x} \longmapsto \sqrt{4+\frac{1}{2}x} = \sqrt{4+\frac{x}{2}}$

horiz. <u>stretch</u> by 2 (reciprocal); x-values times 2
y-values fixed

$\text{dom } f_{S_1} = \text{"}[-4 \cdot 2, \infty \cdot 2)\text{"} = [-8, \infty)$; $\text{rng } f_{S_1} = [0, \infty)$

S2: $\sqrt{4+\frac{x}{2}} \longmapsto \frac{1}{3}\sqrt{4+\frac{x}{2}}$

vertical <u>shrink</u> by $\frac{1}{3}$; x-values fixed
y-values times $\frac{1}{3}$

$\operatorname{dom} f_{S_2} = [-8,\infty)$

$\operatorname{rng} f_{S_2} = \text{"}[0\cdot\frac{1}{3}, \infty\cdot\frac{1}{3})\text{"} = [0,\infty)$

R1: $\frac{1}{3}\sqrt{4+\frac{x}{2}} \longmapsto -\frac{1}{3}\sqrt{4+\frac{x}{2}}$

reflect across x-axis ; x-values fixed
y-values times -1

$\operatorname{dom} f_{R_1} = [-8,\infty)$

$\operatorname{rng} f_{R_1} = \text{"}[(0)(-1), \infty\cdot(-1))\text{"} = \text{"}[0,-\infty)\text{"} = (-\infty,0]$

R2: $-\frac{1}{3}\sqrt{4+\frac{x}{2}} \longmapsto -\frac{1}{3}\sqrt{4-\frac{x}{2}}$

reflect across y-axis ; x-values times -1
y-values fixed

$\operatorname{dom} f_{R_2} = \text{"}[(-8)(-1), \infty\cdot(-1))\text{"} = \text{"}[8,-\infty)\text{"} = (-\infty, 8]$

$\operatorname{rng} f_{R_2} = (-\infty, 0]$

V: $-\frac{1}{3}\sqrt{4-\frac{x}{2}} \longmapsto -\frac{1}{3}\sqrt{4-\frac{x}{2}} + 2$

$= 2 - \frac{1}{3}\sqrt{4-\frac{x}{2}}$

up 2 ; x-values fixed
y-values +2

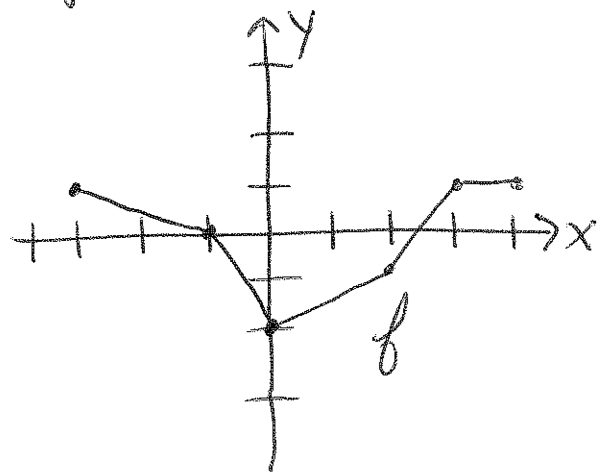
$\operatorname{dom} f_V = (-\infty, 8]$

$\operatorname{rng} f_V = \text{"}(-\infty+2, 0+2]\text{"}$

$= (-\infty, 2]$

Ans: $\boxed{\begin{array}{l} \text{dom } f = (-\infty, 8] \\ \text{rng } f = (-\infty, 2] \end{array}}$ ⟵ Note: Same result as by 2.1/2.2H and I

Example 3: The graph of f is given by



Find dom g and rng g if

$$g(x) = -6 - 3f(-2x-1)$$

Solution

Base Graph: dom $f = [-3, 4]$ (using 2.1/2.2N method)

rng $f = [-2, 1]$

H: $y = f(x) \longmapsto f(x-1)$

right 1; x-values $+1$
y-values fixed

dom $f_h =$ "$[-3+1, 4+1]$" $= [-2, 5$

rng $f_h = [-2, 1]$

S1: $f(x-1) \longmapsto f(2x-1)$

horiz. shrink by $\frac{1}{2}$; x-values times $\frac{1}{2}$
y-values fixed

dom $f_{S_1} =$ "$[-2 \cdot \frac{1}{2}, 5 \cdot \frac{1}{2}]$"
$= [-1, \frac{5}{2}]$

rng $f_{S_1} = [-2, 1]$

S2: $f(2x-1) \longmapsto 3f(2x-1)$

vert. stretch by 3; x-values fixed
y-values times 3

$\text{dom } f_{S_2} = [-1, \frac{5}{2}]$

$\text{rng } f_{S_2} = \text{"}[-2\cdot 3, 1\cdot 3]\text{"} = [-6, 3]$

R1: $3f(2x-1) \longmapsto -3f(2x-1)$

x-axis reflection; x-values fixed
y-values times -1

$\text{dom } f_{R_1} = [-1, \frac{5}{2}]$

$\text{rng } f_{S_1} = \text{"}[-6\cdot(-1), 3\cdot(-1)]\text{"} = \text{"}[6, -3]\text{"} = [-3, 6]$

R2: $-3f(2x-1) \longmapsto -3f(-2x-1)$

y-axis reflection; x-values times -1
y-values fixed

$\text{dom } f_{R_2} = \text{"}[(-1)\cdot(-1), \frac{5}{2}\cdot(-1)]\text{"} = \text{"}[1, -\frac{5}{2}]\text{"} = [-\frac{5}{2}, 1]$

$\text{rng } f_{R_2} = [-3, 6]$

V: $-3f(-2x-1) \longmapsto -3f(-2x-1) - 6$
$= -6 - 3f(-2x-1)$

down 6; x-values fixed; y-values -6

$\text{dom } f_V = [-\frac{5}{2}, 1]$, $\text{rng } f_V = \text{"}[-3-6, 6-6]\text{"}$
$= [-9, 0]$

<u>Ans</u>: $\boxed{\text{dom } g = [-\frac{5}{2}, 1] \;;\; \text{rng } g = [-9, 0]}$