2.5C HSRV I: Output Formulas

A. HSRV

A given function may be constructed from a combination of transformations. To draw the graph, we transform the base graph in the following order:

1. Horizontal Translations
2. Stretching/Shrinking [Horizontal and Vertical]
3. Reflections [x-axis and y-axis]
4. Vertical Translations

If you do not follow this order, you will not get the correct final graph! Reason for order: function composition [Discussed in Section 2.6]

B. Method

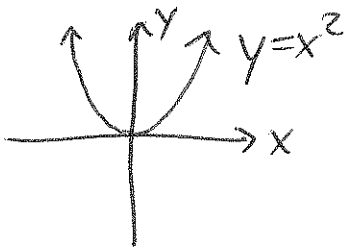
1. Identify which of the six common base graphs is "buried" inside the formula. Draw a rough sketch of it.
2. Perform the transformations, one at a time, in the proper order to the base graph drawing a new rough picture.
3. Find the x & y intercepts
4. Use the x & y intercepts with final rough picture to draw the final graph.

C. Examples

Example1: Graph f, where $f(x) = 3-(x+2)^2$

Solution

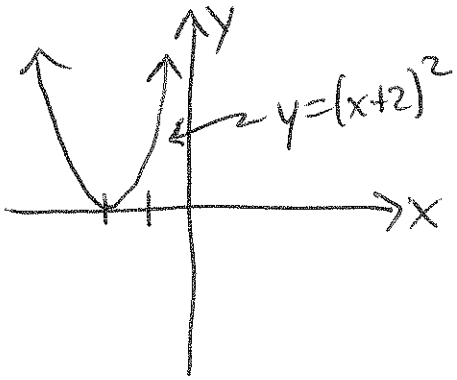
1. Identify base graph $y=x^2$:



2. Do HSRV:

a. **Horizontal Translation** : $x^2 \longmapsto (x+2)^2$

add 2 inside, left 2, x-values -2 (opposite), y-values fixed

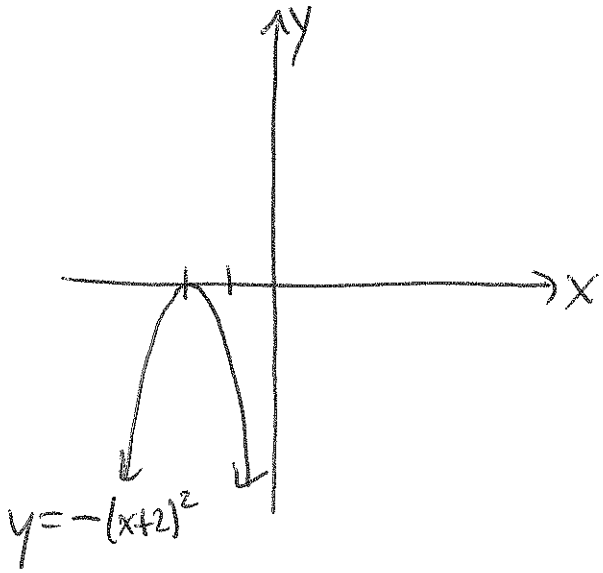


b. **Stretching/Shrinking** : none

(neither outside nor x is multiplied)

c. **Reflections**: $(x+2)^2 \longmapsto -(x+2)^2$

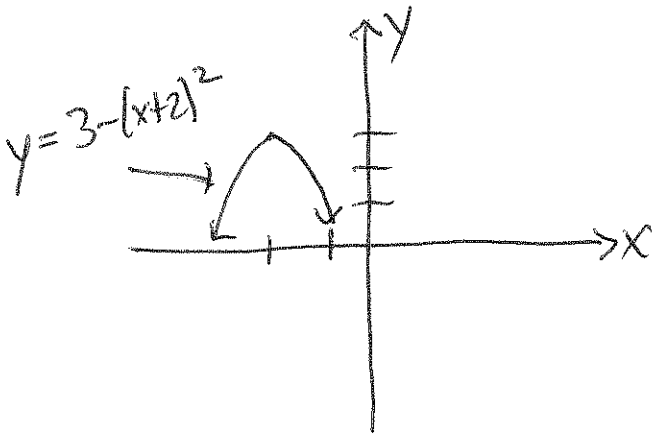
outside $-$ sign : x-axis reflection
x-values fixed
y-values times -1



d. Vertical Translation: $-(x+2)^2 \longmapsto -(x+2)^2+3$

$= 3-(x+2)^2$

add 3 outside, up 3, x-values fixed
y-values +3



3. Find x and y intercepts:

x-intercepts: set $y=0$: $0 = 3-(x+2)^2$

$(x+2)^2 = 3$

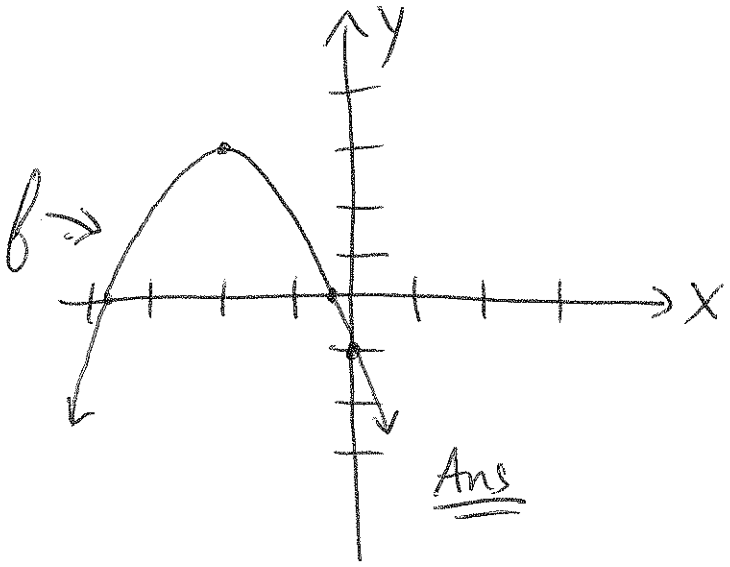
$\Rightarrow x+2 = \pm\sqrt{3}$

$x = -2 \pm \sqrt{3}$ $\begin{cases} \approx -.268 \\ \approx -3.73 \end{cases}$

y-intercept: set $x=0$: $y = 3-(0+2)^2$

$= 3-4 = -1$

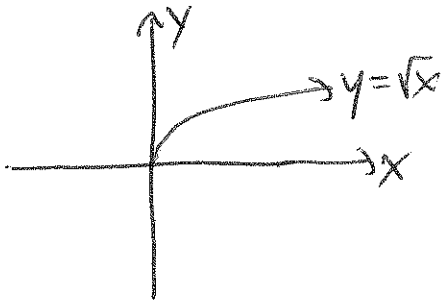
4.



Example 2: Graph g, where $g(x) = \frac{7}{2}\sqrt{-x} - 3$

Solution

1. Identify base graph: $y = \sqrt{x}$:



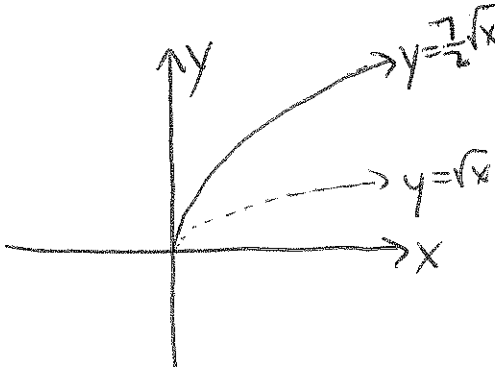
2. Do HSRV:

a. Horizontal Translation: none
(no adding/subtracting inside $\sqrt{\ }$)

b. Stretching/Shrinking: $\sqrt{x} \longmapsto \frac{7}{2}\sqrt{x}$

outside multiplier $\frac{7}{2}$: vertical stretch by $\frac{7}{2}$

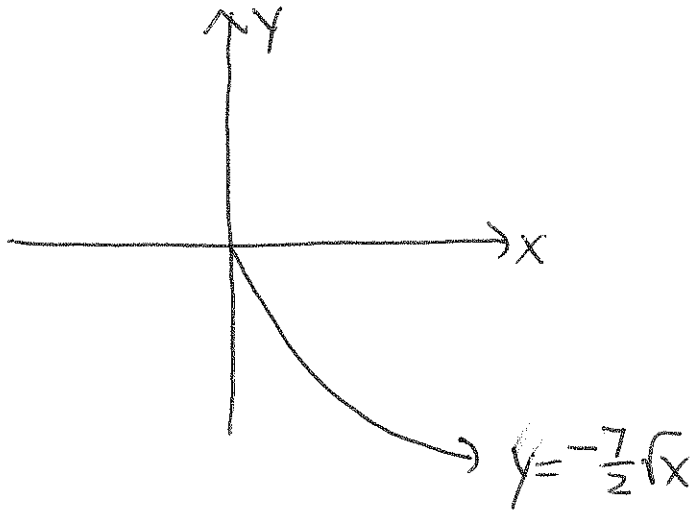
x-values fixed
y-values multiplied by $\frac{7}{2}$



c. Reflections:

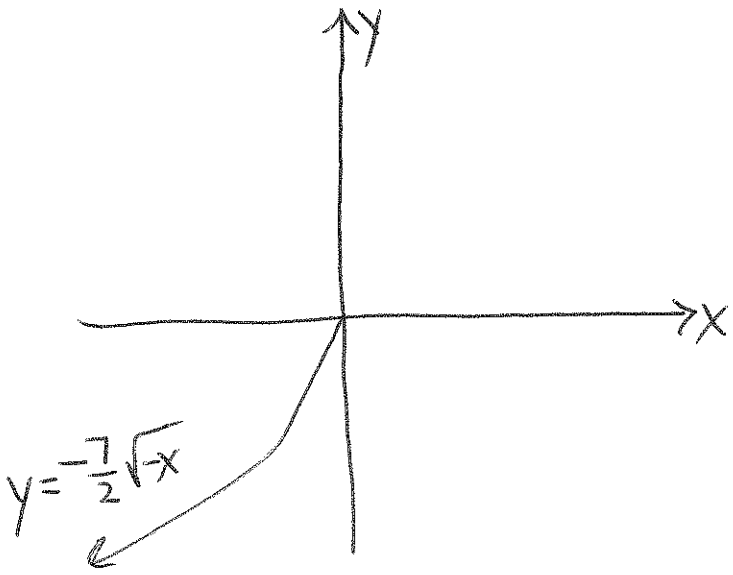
I. $\frac{7}{2}\sqrt{x} \longmapsto -\frac{7}{2}\sqrt{x}$

outside $-$: x-axis reflection
x-values fixed
y-values times -1



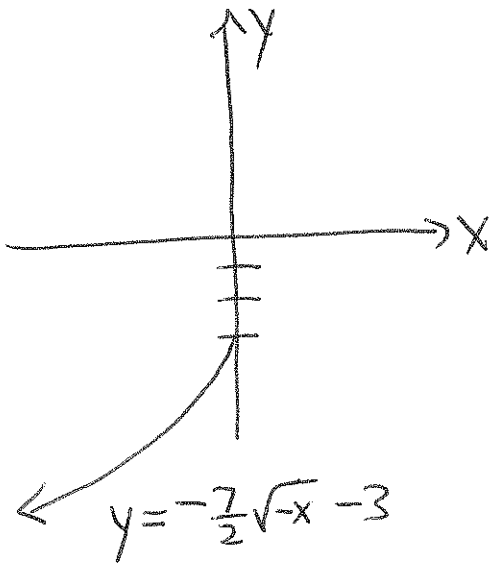
II. $-\frac{7}{2}\sqrt{x} \longmapsto -\frac{7}{2}\sqrt{-x}$

inside $-$: y-axis reflection
x-values times -1
y-values fixed



d. Vertical Translation: $-\frac{7}{2}\sqrt{-x} \longmapsto -\frac{7}{2}\sqrt{-x}-3$

subtract 3 outside, down 3, x-values fixed
y-values -3



3. Find x & y intercepts

x-intercepts: set $y=0$: $0=-\frac{7}{2}\sqrt{-x}-3$

$$\frac{7}{2}\sqrt{-x}=-3$$

$$7\sqrt{-x}=-6$$

$$\sqrt{-x}=-\frac{6}{7}$$

$$-x=\left(-\frac{6}{7}\right)^2=\frac{36}{49}$$

$$x=\frac{-36}{49}$$

Mandatory Check: $0\stackrel{?}{=}-\frac{7}{2}\sqrt{\frac{36}{49}}-3$

$0\stackrel{?}{=}-\frac{7}{2}\left(\frac{6}{7}\right)-3$

$0\stackrel{?}{=}-3-3$ X

no x-intercepts!

y-intercept: set $x=0$:

$$y=-\frac{7}{2}\sqrt{-0}-3=-\frac{7}{2}(0)-3=-3$$

4.