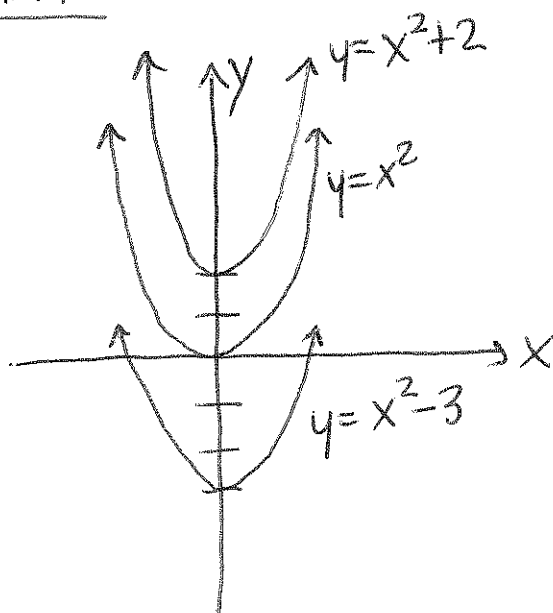


2.5B Basic Transformations

A. Vertical Shifts

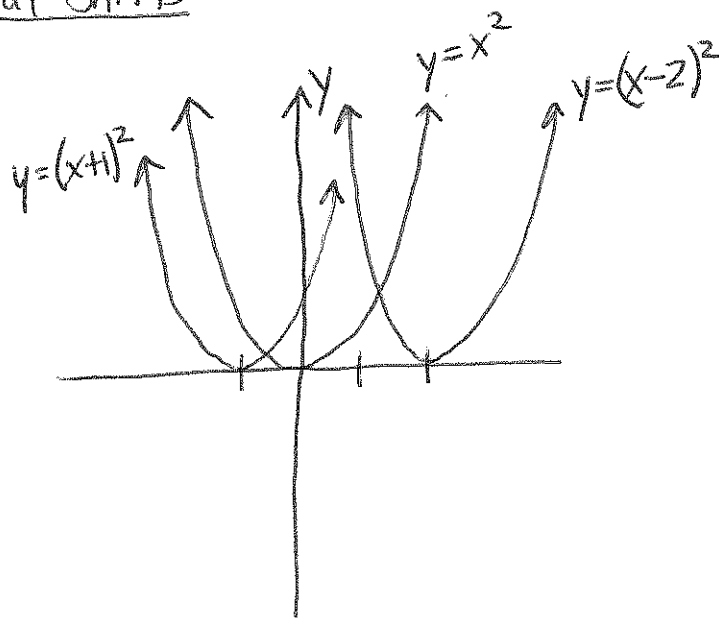


1. Rule: Adding/subtracting a number OUTSIDE of $f(x)$ moves graph UP/DOWN

2. Notation: $f(x) \mapsto f(x) \pm c$

3. Effect on Coordinates: x-values are fixed
y-values add/subtract
(useful for general transformations later)

B. Horizontal Shifts



1. Rule: Adding/subtracting a number INSIDE of $f(x)$ moves graph LEFT/RIGHT

2. Notation: $f(x) \mapsto f(x \pm c)$

3. Effect on Coordinates: x-values add/subtract OPPOSITELY
y-values are fixed

(useful for general transformations)

4. Warning About "oppositely":

a. Adding a number INSIDE

① Moves graph LEFT

② x-values SUBTRACT

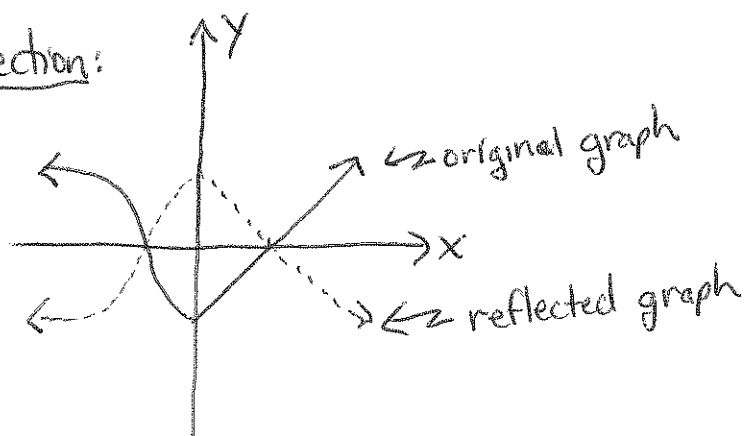
b. Subtracting a number INSIDE

① Moves graph RIGHT

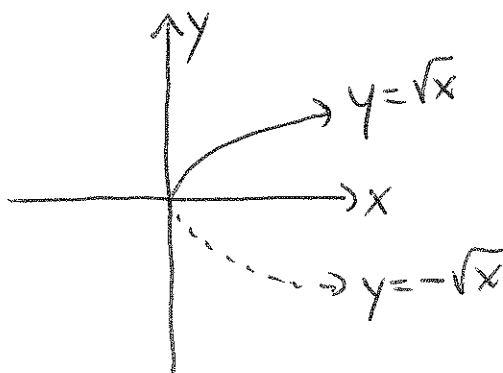
② x-values ADD

C. Reflections

1. x-axis reflection:



a. Rule: Multiplying $f(x)$ by -1 , reflects across the x-axis

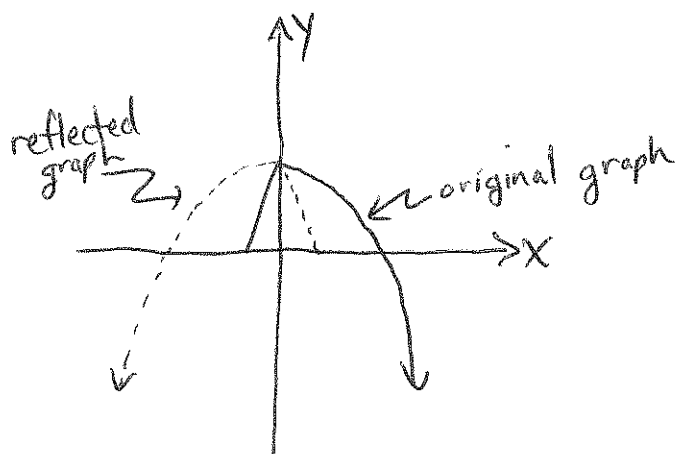


b. Notation: $f(x) \mapsto -f(x)$

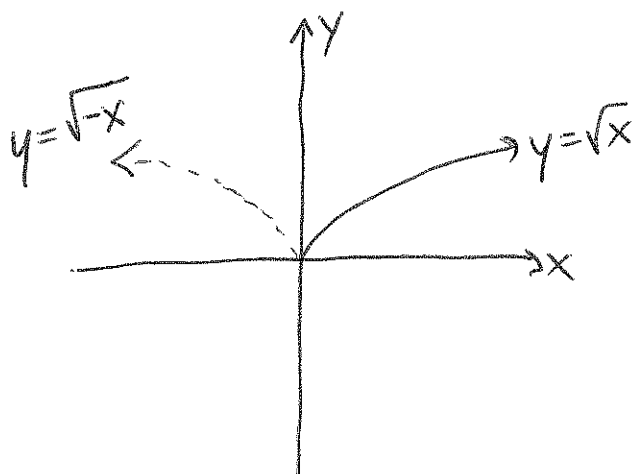
c. Effect on Coordinates: x-values are fixed
y-values multiplied by -1

(useful for general transformations)

2. y-axis reflection:



a. Rule: Replacing x with " $-x$ " reflects the graph across y-axis.



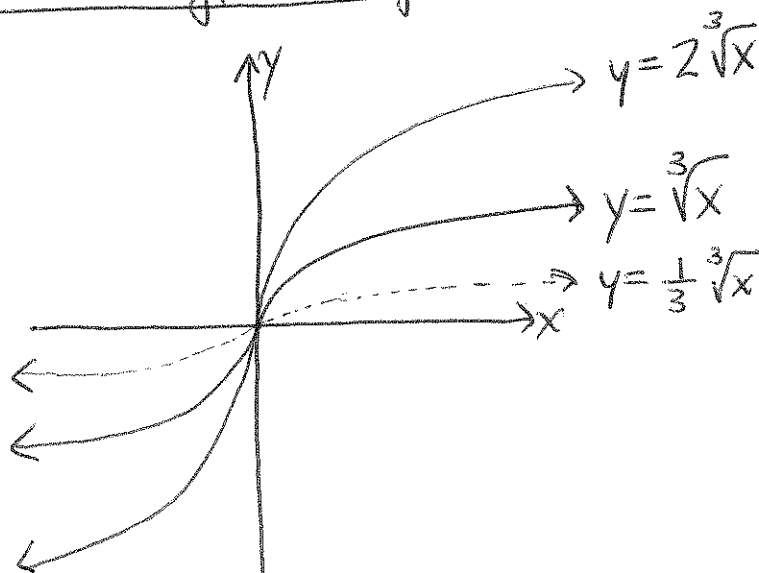
b. Notation: $f(x) \mapsto f(-x)$

c. Effect on Coordinates: x -values are multiplied by -1
 y -values are fixed

(useful for general transformations)

0. Stretching/Shrinking

1. Vertical Stretching/Shrinking



a. Rule: Multiplying $f(x)$ by a positive number stretches/shrinks the graph vertically.

b. Notation: $f(x) \mapsto c f(x)$ [$c > 1$ stretch]

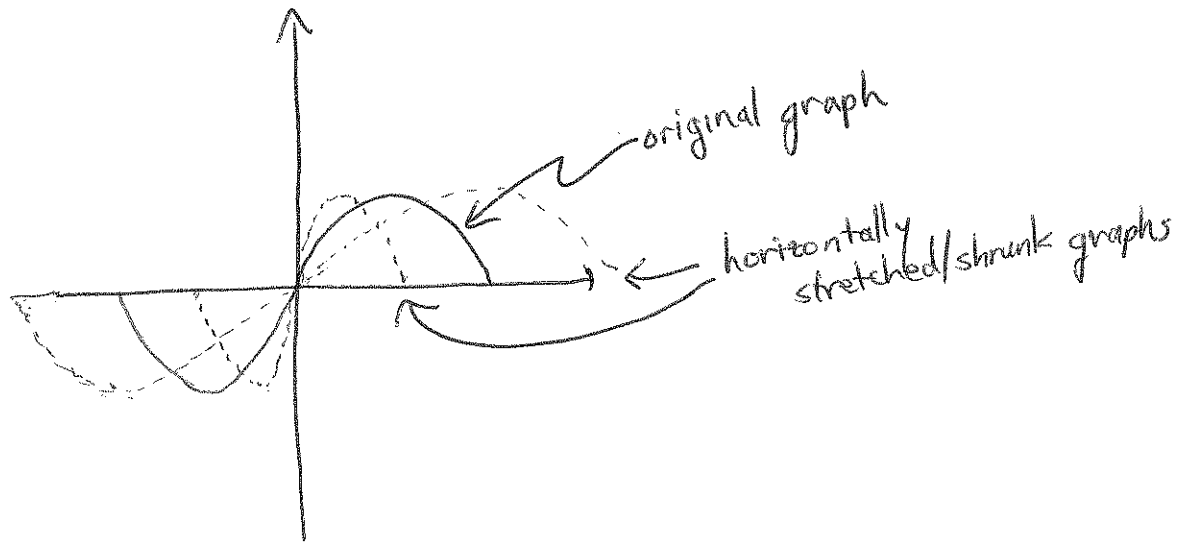
c. Effect on Coordinates: x-values are fixed
y-values get multiplied

(useful for general transformations)

d. Comment on Negative Factors: if the outside is multiplied by a "negative number", we think of the vertical stretching/shrinking factor as the positive number. Here you have stretching/shrinking followed by a reflection. More on this later

ie. $y = -2\sqrt[3]{x}$ $\leftarrow$ vertical stretch/shrink by 2
then do an x-axis reflection!

2. Horizontal Stretching/Shrinking:



a. Rule: Multiplying x on the INSIDE of $f(x)$ by a positive number stretches/shrinks the graph horizontally, but does so oppositely by reciprocal

b. Notation: $f(x) \mapsto f(cx)$ $[c > 1$ shrink]

c. Effect on Coordinates: x -values get multiplied by RECIPROCAL
 y -values are fixed

(useful for general transformations)

d. Comment on Negative Factors: Again, we separate off the minus sign and consider the multiplying factor to be positive. Thus the minus sign is a follow-up reflection.

e. Warning About RECIPROCAL:

$f(3x) \leftarrow x$ -values get multiplied by $\frac{1}{3}$

$f(\frac{1}{2}x) \leftarrow x$ -values get multiplied by 2

E. Comments

1. Horizontal and vertical shifts are called translations.
2. Be careful with horizontal effects.
Movements and x-coordinates act "counterintuitively".
3. All of the actions and information in this section needs to be memorized. In the next section, these will be put together in examples containing more than one transformation!