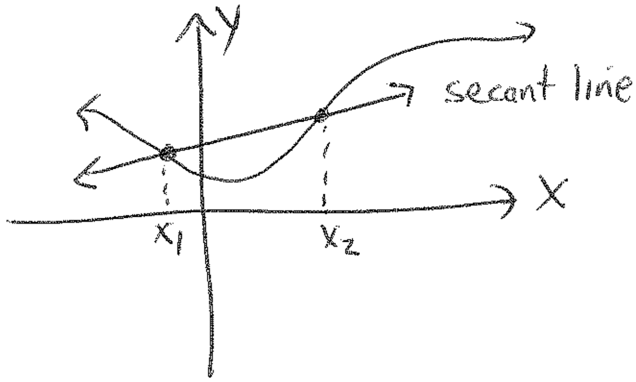
2.4B Average Rate of Change

A. Secant Lines

A line that cuts through a function in at least two places is called a secant line.



B. Average Rate of Change

The average rate of change of a function from x-values x_1 to x_2 is defined to be the slope of the corresponding secant line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

← since y_1 and y_2 are what you get when you plug x_1 and x_2 into the function.

$$\boxed{\frac{f(x_2) - f(x_1)}{x_2 - x_1}}$$

Average Rate of Change

C. Example

Let $f(x) = x^2 + 3x - 2$

Find the average rate of change of f from $x_1 = -1$ to $x_2 = 3$

Solution

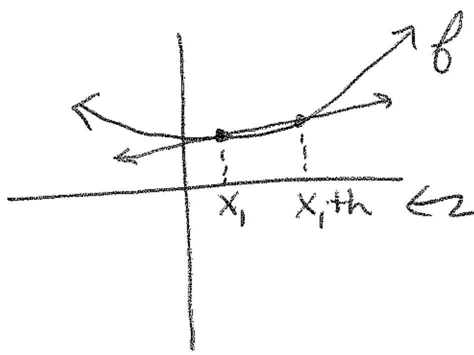
$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(3) - f(-1)}{3 - (-1)} = \frac{f(3) - f(-1)}{4}$$

Now $f(3) = 3^2 + 3(3) - 2 = 9 + 9 - 2 = 16$

and $f(-1) = (-1)^2 + 3(-1) - 2 = 1 - 3 - 2 = -4$

Thus we have $\frac{16 - (-4)}{4} = \frac{20}{4} = \boxed{5}$ Ans

D. Slope of Secant Lines for Near Points



$\Leftarrow$ assume x_2 is "just a little more" than x_1

Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1 + h) - f(x_1)}{(x_1 + h) - x_1}$

$$m = \frac{f(x_1 + h) - f(x_1)}{h}$$

Thus the difference quotient $\frac{f(x+h) - f(x)}{h}$ is a formula for the slope of all secant lines with first point "x".