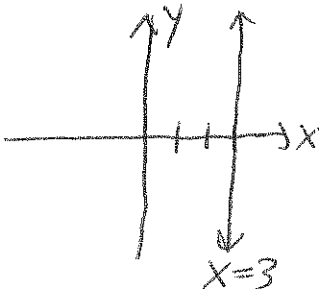
2.3/2.4 Review of Lines and Slope

A. Lines

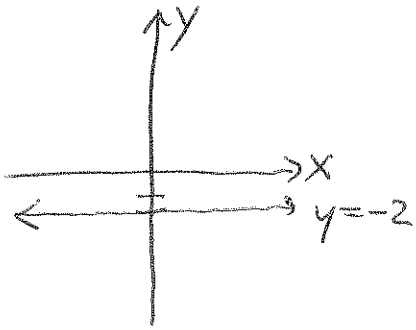
A line is represented by an equation in x and y to only first degree powers.

B. Graphing Lines

1. A vertical line has the equation $x=c$



2. A horizontal line has the equation $y=d$

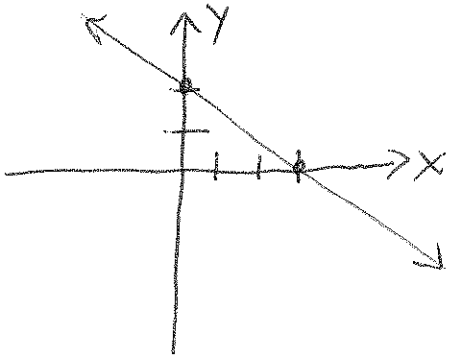


3. Angled Lines: Find the x and y intercepts

Ex) $2x+3y=6$

x-int: set $y=0$: $2x=6 \Rightarrow x=3$

y-int: set $x=0$: $3y=6 \Rightarrow y=2$

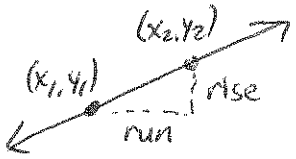


Note: If both x and y intercepts are 0 (line passing through origin), you'll need another point! Just pick an x value and plug in to get a new point.

C. Slope

Measures how much the line is "tilting"; Represented by m

1. Horizontal Lines: $m=0$
2. Vertical Lines: m is undefined
3. Angled Lines: $m = \frac{\text{rise}}{\text{run}} = \frac{y_2-y_1}{x_2-x_1}$



D. Equations of Lines

1. Horizontal Lines: $y=d$
2. Vertical Lines: $x=c$
3. Angled Lines:
   - I. Point-Slope Form: $y-y_1=m(x-x_1)$ ↔ most useful for solving problems
   - II. Slope-Intercept Form: $y=mx+b$ — b: y-intercept
   - III. General Form: $Ax+By+C=0$

E. Finding the Equation of a Line

Method:

1. First find the slope of the given line
   - I. If the line is horizontal, then $m=0$
   - II. If the line is vertical, then m is undefined
   - III. If you have an angled line, we have different methods ...

Methods for finding the slope of an angled line:

A. Given a graph, use $m = \frac{\text{rise}}{\text{run}}$

B. Given two points, use $m = \frac{y_2 - y_1}{x_2 - x_1}$

C. Given point-slope or slope-intercept form: read off m

D. Given general form $Ax + By + C = 0$: $m = \frac{-A}{B}$

2. Find the slope of the line you want

I. Nothing different: use answer from #1

II. Parallel Line:

A. If $m = 0$, then $m_{\parallel} = 0$

B. If m is undefined, then $m_{\parallel}$ is undefined

C. In any other case, $m_{\parallel} = m$ (same slope)

III. Perpendicular Line:

A. If $m = 0$, then $m_{\perp}$ is undefined

B. If m is undefined, then $m_{\perp} = 0$

C. In any other case, $m_{\perp} = \frac{-1}{m}$ (negative reciprocal)

3. Getting the equation

Using the slope from step #2:

I. If $m = 0$, then equation is $y = d$ (horizontal line)

II. If m is undefined, then equation is $x = c$ (vertical line)

III. In any other case, write down the point-slope form:

$$y - y_1 = m(x - x_1)$$

4. Having the answer in a particular form

You are already done unless the problem asks for slope-intercept form or general form.

I. If you need slope-intercept form: solve the equation for y

II. If you need general form:

A. First find slope-intercept form by solving for y

B. Move everything to one side

C. For a nice answer, clear fractions making the coefficient for x positive

F. Examples

Example 1: Find the equation of the line with slope -2 passing through $(-3,4)$. Write the equation in general form

Solution

Steps 1 and 2 ✓

Step 3: Point-Slope: $y-y_1=m(x-x_1)$

$y-4=-2(x+3)$ ← $x-(-3)$

Step 4: ① Solve for y: $y-4=-2x-6$

$y=-2x-2$

② Move everything to one side:

$$\boxed{2x+y+2=0}$$ Ans

Example 2: Find the equation of the line passing through $(-1,4)$ and $(-1,7)$

Solution

Step 1: Find slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{(-1)-(-1)} = \frac{3}{0}$ undefined

Step 2: ✓

Step 3: Equation is $x = c$ (x-coordinate)

$\boxed{x = -1}$ Ans

Example 3: Find the equation of the line passing through $(-3,2)$ and $(1,4)$. Write your answer in slope-intercept form.

Solution

Step 1: Find slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{1-(-3)} = \frac{2}{4} = \frac{1}{2}$

Step 2: ✓

Step 3: Point-Slope Form: $y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{2}(x+3)$

Step 4: Solve for y: $y - 2 = \frac{1}{2}x + \frac{3}{2}$

$y = \frac{1}{2}x + \frac{3}{2} + 2$

$2 = \frac{4}{2}$ ↳

$\boxed{y = \frac{1}{2}x + \frac{7}{2}}$ Ans

Example4: Find the equation of the line passing through $(1,-3)$ and perpendicular to $3x-2y-5=0$. Write your answer in general form.

Solution

Step 1: Find slope: $m=\frac{-A}{B}=\frac{-3}{-2}=\frac{3}{2}$

Step 2: Perpendicular slope: $m_{\perp}=\frac{-2}{3}$

Step 3: Point-slope: $y-y_1=m(x-x_1)$

$y+3=-\frac{2}{3}(x-1)$ (using perpendicular slope!)

Step 4: ① Solve for y: $y+3=-\frac{2}{3}x+\frac{2}{3}$

$y=-\frac{2}{3}x+\frac{2}{3}-3 \quad \leftarrow 3=\frac{9}{3}$

$y=-\frac{2}{3}x-\frac{7}{3}$

② Move every thing to one side:

$\frac{2}{3}x+y+\frac{7}{3}=0$

③ Clear fractions (keeping x coefficient positive):

$3\left(\frac{2}{3}x+y+\frac{7}{3}\right)=3\cdot 0$

$\boxed{2x+3y+7=0}$ Ans

Example 5: Find the equation of the line passing through $(-2,4)$ that is perpendicular to the line passing through $(3,-4)$ and $(3,7)$

Solution

Step 1: Find slope of given line: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-(-4)}{3-3} = \frac{11}{0}$

m is undefined

Step 2: Perpendicular slope: $m_\perp = 0$

Step 3: Equation is $y = d$ (y-coordinate of line you want)

$\boxed{y = 4}$ Ans

Example 6: Find the equation of the line passing through $(2,-1)$ that is parallel to the line $y = \frac{1}{3}x + 2$. Write your answer in general form.

Solution

Step 1: $m = \frac{1}{3}$

Step 2: $m_{\parallel} = \frac{1}{3}$ (same slope)

Step 3: Point-Slope: $y - y_1 = m(x - x_1)$

$y + 1 = \frac{1}{3}(x - 2)$ [using parallel slope]

Step 4: ① Solve for y: $y + 1 = \frac{1}{3}x - \frac{2}{3}$

$y = \frac{1}{3}x - \frac{2}{3} - 1$

$y = \frac{1}{3}x - \frac{5}{3}$

② Move everything to one side: $-\frac{1}{3}x + y + \frac{5}{3} = 0$

③ Clear fractions (making x coeff pos.):

$-3\left(-\frac{1}{3}x + y + \frac{5}{3}\right) = -3 \cdot 0 \Rightarrow \boxed{x - 3y - 5 = 0}$ Ans