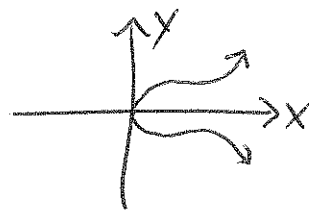


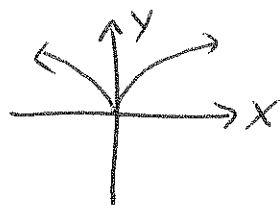
2.1/2.2 Q Symmetry; Even and Odd

A. Symmetry

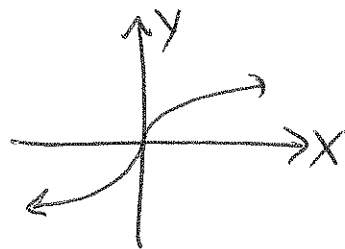
1. x-axis: x-axis looks like a mirror



2. y-axis: y-axis looks like a mirror



3. origin: origin looks like a mirror



Note: A nonzero function can't have x-axis symmetry,
[vertical line test]

B. Symmetry of Functions

1. A function with y-axis symmetry is called even.

2. A function with origin symmetry is called odd.

Note: A function may have no symmetry (y-axis/origin)
Then it is said to be neither even nor odd.

C. Even/Odd Tests

1. A function is even if

a. informal: replacing x with $-x$ keeps the function output formula unchanged

b. formal: $f(-x) = f(x)$

2. A function is odd if

a. informal: replacing x with $-x$ and replacing y with $-y$ keeps the function output formula unchanged,

b. formal: $f(-x) = -f(x)$

D. Using Even/Odd Tests

1. To test to see if a function is even/odd, compute $f(-x)$. If this is the same as $f(x)$, f is even; if it is the same as $-f(x)$, f is odd. Otherwise f is neither.

2. When testing, you must use generic x , not just one point. For example, given $f(x) = x^3 - x - 2$, we see that $f(1) = -2$ and $f(-1) = -2$. However, f is not even. It must be true for all x .

E. Examples

Example 1: Determine if f is even, odd, or neither when $f(x) = x^3$.

Solution

Compute $f(-x)$ and compare to $f(x)$ and $-f(x)$:

$$f(-x) = (-x)^3 = (-1)^3 x^3 = -x^3$$

$$f(x) = x^3$$

$$-f(x) = -x^3$$

Since $f(-x) = -f(x)$, f is odd Ans

Example 2: Determine if g is even, odd, or neither when $g(x) = |x|$.

Solution

Compute $g(-x)$ and compare to $g(x)$ and $-g(x)$

$$g(-x) = |-x| = |-1||x| = |x|$$

$$g(x) = |x|$$

$$-g(x) = -|x|$$

Since $g(-x) = g(x)$, g is even Ans

Example 3: Determine if h is even, odd, or neither when $h(x) = (x-1)^2$.

Solution

Compute $h(-x)$ and compare to $h(x)$ and $-h(x)$:

$$h(-x) = (-x-1)^2 = x^2 + 2x + 1$$

$$h(x) = (x-1)^2 = x^2 - 2x + 1$$

$$-h(x) = -(x-1)^2 = -(x^2 - 2x + 1) = -x^2 + 2x - 1$$

Since $h(-x)$ is not equal to either $h(x)$ or $-h(x)$,
 h is neither even nor odd.

Ans