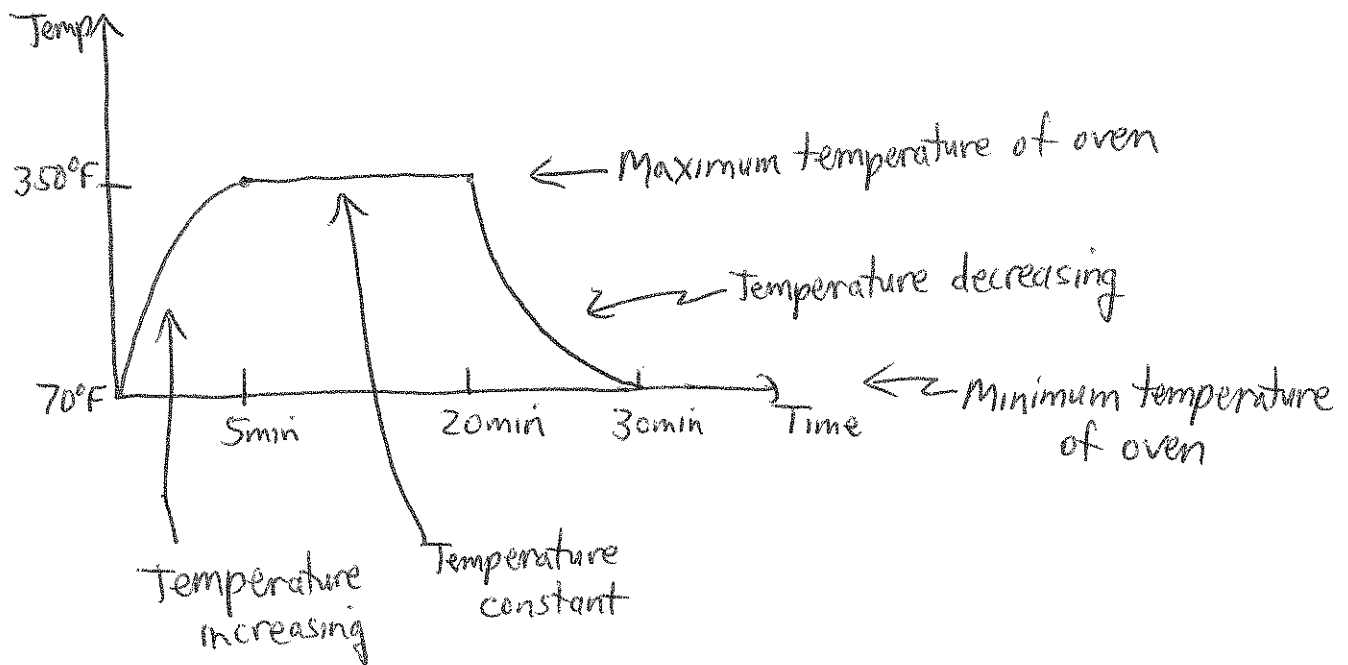


2.1/2.2 P Increasing and Decreasing Functions; Extreme Values

A. Motivating Model: Baking Cookies

Preheat oven, bake cookies, turn oven off
Graph temperature as a function of time



Observations:

- Temp increasing from (0, 5) [min]
- Temp constant from (5, 20) [min]
- Temp decreasing from (20, 30) [min]
- Maximum temperature: 350°F
- Minimum temperature: 70°F

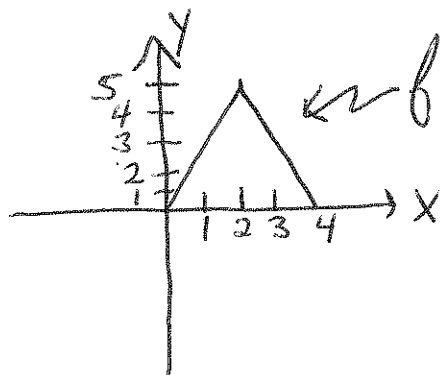
From this idea, we can develop rules
for graph behavior.

B. Graph Behavior: Informal Description

1. Increasing/decreasing: graph goes up/down as you look from left to right, and we describe with intervals along the x-axis using x-values.
2. Constant: graph is "horizontal"; again we describe with intervals along the x-axis using x-values.
3. Maximum/minimum: the highest/lowest point on the graph, and we report the y-value (output)

Note: The convention used here is that intervals for increasing/decreasing/constant are always open, i.e. $(,)$ not $[,]$

4. Preliminary Example:



We see that f is

- increasing on $(0,2)$
- decreasing on $(2,4)$
- has a maximum of 5
- has a minimum of 0

C. Comments on Extreme Values

1. Maxima/minima are called extreme values or extrema

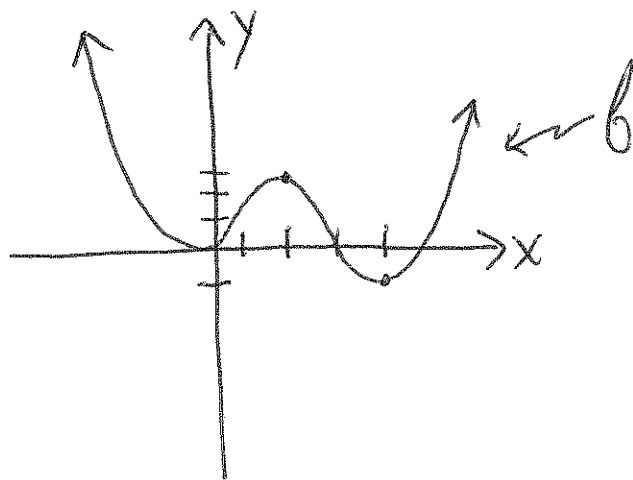
2. Extrema come in two types

a. relative/local: max or min in local area
"peak" or "valley"

b. absolute/global: "overall" max or min

3. There can be only one absolute/global max or min,
but many possible relative/local maxs or mins.

4. Example:



For f : relative minima: 0 and -1
relative maximum: $5/2$
absolute minimum: -1
absolute maximum: none!

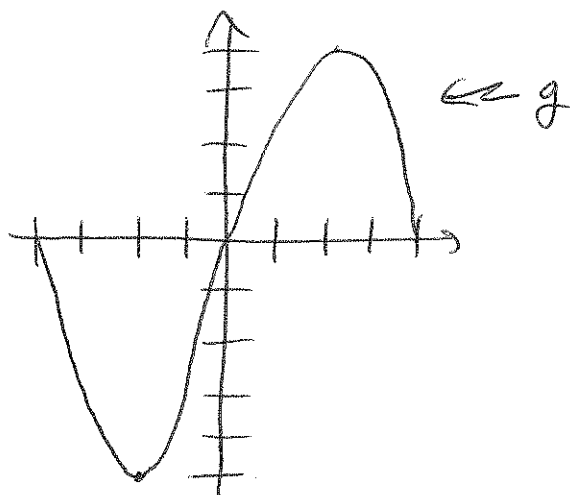
↗ y-values

increasing on $(0, 2) \cup (4, \infty)$
decreasing on $(-\infty, 0) \cup (2, 4)$

↗ x-intervals

D. Examples

Example 1: The graph of a function g is shown in the figure:



- State the values of $g(0)$ and $g(1)$
- State the intervals on which g is increasing and on which g is decreasing.
- State the minimum value of g . At what value of x does this occur?
- State the maximum value of g . At what value of x does this occur?
- If g is given as above, find $\text{dom } g$ and $\text{rng } g$

Solution

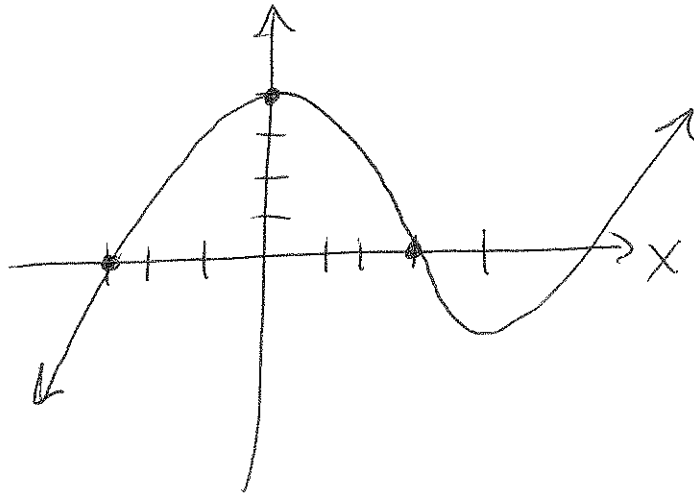
- By inspection, $g(0) = 0$ and $g(1) = 3$
- g is increasing on $(-2, 2)$
 g is decreasing on $(-4, -2) \cup (2, 4)$
- minimum: -5 (occurs at $x = -2$)
- maximum: 4 (occurs at $x = 2$)
- $\text{dom } g = [-4, 4]$ and $\text{rng } g = [-5, 4]$

Example 2: Sketch the function f where f is increasing on $(-\infty, 0) \cup (4, \infty)$, decreasing on $(0, 4)$, and $f(-3) = f(3) = 0$, $f(0) = 4$

Solution

Plot Points: $(-3, 0)$, $(3, 0)$, $(0, 4)$

Then fill in increasing/decreasing behavior



Ans