

2.1/2.2 J Function Equality

A. Conditions

Two functions are equal if two conditions are met:

1. They produce the same output. Thus $f(x) = g(x)$
2. They have the same domain. Thus $\text{dom } f = \text{dom } g$

B. Examples

Example 1: Is $f = g$ where $f(x) = x^3 + 5$ and $g(x) = 4 - x^2$?

Solution

1. Is $f(x) = g(x)$? No

Ans: $f \neq g$ (No)

Example 2: Is $f = g$ where $f(x) = x + 1$ and $g(x) = \frac{(x+1)(x-1)}{x-1}$?

Solution

1. Is $f(x) = g(x)$?

$$f(x) = x + 1$$

$$g(x) = \frac{(x+1)(x-1)}{x-1} = x + 1 \quad (\text{canceling})$$

Thus $f(x) = g(x)$ ✓

2. Is $\text{dom } f = \text{dom } g$?

$$\text{dom } f = (-\infty, \infty)$$

$$\text{dom } g = (-\infty, 1) \cup (1, \infty) \quad [x=1 \text{ is division by zero}]$$

So $\text{dom } f \neq \text{dom } g \Rightarrow \text{Ans: } \boxed{f \neq g} \text{ (No)}$

Example 3: Is $f=g$, where $f(x)=\sqrt{(x+1)^2}$ and $g(x)=|x+1|$?

Solution

1. Is $f(x)=g(x)$?

$$f(x)=\sqrt{(x+1)^2}=|x+1| \quad \left(\text{using piecewise formula for } \sqrt[n]{x^n} \right)$$

so $f(x)=g(x)$ ✓

2. Is $\text{dom } f = \text{dom } g$?

$\text{dom } f$: set inside ≥ 0

$$(x+1)^2 \geq 0$$



$$\text{dom } f = (-\infty, \infty)$$

$$\text{dom } g: \text{dom } g = (-\infty, \infty)$$

so $\text{dom } f = \text{dom } g$ ✓

Both conditions are met, so $\boxed{f=g \text{ (YES)}}$