2.1/2.2 G Difference Quotients

A. Difference Quotients

Expressions like $\frac{f(x+h)-f(x)}{h}$ are called difference quotients.

They arise in calculus.

B. Evaluating Difference Quotients

Evaluate each piece with f separately.
Then simplify using standard algebra techniques (MTH1825)

C. Examples

Example1: Let $f(x)=2x^2-x$. Find $\frac{f(x+h)-f(x)}{h}$ and simplify

Solution

$$f(x+h)=2(x+h)^2-(x+h)=2(x^2+2xh+h^2)-x-h$$
$$=2x^2+4xh+2h^2-x-h$$

$$f(x)=2x^2-x$$

Then $$\frac{f(x+h)-f(x)}{h}=\frac{(2x^2+4xh+2h^2-x-h)-(2x^2-x)}{h}$$

$$=\frac{4xh+2h^2-h}{h}=\frac{h(4x+2h-1)}{h}$$

$$=\boxed{4x+2h-1;\ h\neq 0}$$ Ans

Example2: Let $f(x) = 3x^2 - 4x + 1$

Find $\frac{f(2+h) - f(2-h)}{2h}$ and simplify

Solution

$$f(2+h) = 3(2+h)^2 - 4(2+h) + 1$$
$$= 3(4 + 4h + h^2) - 8 - 4h + 1$$
$$= 12 + 12h + 3h^2 - 8 - 4h + 1$$
$$= 5 + 8h + 3h^2$$

$$f(2-h) = 3(2-h)^2 - 4(2-h) + 1$$
$$= 3(4 - 4h + h^2) - 8 + 4h + 1$$
$$= 12 - 12h + 3h^2 - 8 + 4h + 1$$
$$= 5 - 8h + 3h^2$$

Then $\frac{f(2+h) - f(2-h)}{2h} = \frac{(5 + 8h + 3h^2) - (5 - 8h + 3h^2)}{2h}$

$$= \frac{16h}{2h} = \boxed{8;\ h \neq 0}$$ Ans

Note: $\frac{f(x+h) - f(x)}{h}$ is the "standard" difference quotient.

If a problem asks for the difference quotient, it is this expression they want.

Example 3: Let $f(x)=\frac{1}{2x-3}$. Find the difference quotient and simplify.

Solution

We want $\frac{f(x+h)-f(x)}{h}$;

$$f(x+h)=\frac{1}{2(x+h)-3}=\frac{1}{2x+2h-3}$$

$$f(x)=\frac{1}{2x-3}$$

Then $\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{2x+2h-3}-\frac{1}{2x-3}}{h}$ Find a common denominato

$$=\frac{\frac{2x-3}{(2x+2h-3)(2x-3)}-\frac{2x+2h-3}{(2x+2h-3)(2x-3)}}{h}$$

$$=\frac{\frac{-2h}{(2x+2h-3)(2x-3)}}{h}$$

$$=\frac{-2\cancel{h}}{(2x+2h-3)(2x-3)}\cdot\frac{1}{\cancel{h}}$$

$$=\boxed{\frac{-2}{(2x+2h-3)(2x-3)};\ h\neq 0}$$