

2.1/2.2 F Several Copies of f

A. Introduction

When f occurs more than once, do each piece separately.
Then combine the individual pieces.

B. Examples

Example 1: Let $f(x) = 2x^2 - 3x + 1$

Find $f(x+h) - f(x-h)$ and simplify.

Solution

① First find $f(x+h)$:

$$\begin{aligned} f(x+h) &= 2(x+h)^2 - 3(x+h) + 1 \\ &= 2(x^2 + 2xh + h^2) - 3x - 3h + 1 \\ &= 2x^2 + 4xh + 2h^2 - 3x - 3h + 1 \end{aligned}$$

② Now find $f(x-h)$:

$$\begin{aligned} f(x-h) &= 2(x-h)^2 - 3(x-h) + 1 \\ &= 2(x^2 - 2xh + h^2) - 3x + 3h + 1 \\ &= 2x^2 - 4xh + 2h^2 - 3x + 3h + 1 \end{aligned}$$

③ To find $f(x+h) - f(x-h)$, use ① and ②:

$$\begin{aligned} &f(x+h) - f(x-h) \\ &= (2x^2 + 4xh + 2h^2 - 3x - 3h + 1) - (2x^2 - 4xh + 2h^2 - 3x + 3h + 1) \\ &\quad \text{①} \qquad \qquad \qquad \text{②} \\ &= \boxed{8xh - 6h} \text{ Ans} \end{aligned}$$

Example 2: Let $f(x) = \frac{4}{3x+2}$. Find $f(x+h) - f(3x)$ and simplify.

Solution

① First find $f(x+h)$:

$$f(x+h) = \frac{4}{3(x+h)+2} = \frac{4}{3x+3h+2}$$

② Now find $f(3x)$:

$$f(3x) = \frac{4}{3(3x)+2} = \frac{4}{9x+2}$$

③ To find $f(x+h) - f(3x)$, use ① and ②:

$$f(x+h) - f(3x)$$

$$= \frac{4}{3x+3h+2} - \frac{4}{9x+2}$$

① ②

Now get a common denominator:

$$= \frac{4(9x+2)}{(3x+3h+2)(9x+2)} - \frac{4(3x+3h+2)}{(3x+3h+2)(9x+2)}$$

$$= \frac{36x+8}{(3x+3h+2)(9x+2)} - \frac{12x+12h+8}{(3x+3h+2)(9x+2)}$$

$$= \frac{24x-12h}{(3x+3h+2)(9x+2)} = \boxed{\frac{12(2x-h)}{(3x+3h+2)(9x+2)}} \quad \underline{\text{Ans}}$$