1.7B Absolute Value Inequalities

A. Strategy

1. Draw a picture on the number line representing distance.
2. Use the shaded number line to rewrite the problem without absolute values.

Typically, you will end up with two inequalities joined by AND or OR

Tip:  "sandwich" $\Rightarrow$ AND
often occurs with $<, \leq$

 "two pieces outward" $\Rightarrow$ OR
often occurs with $>, \geq$

3. Simplify the AND or OR if present.

Note: There are many, many different cases and possibilities here depending on whether the right hand side is positive, negative, or zero and depending on the type of inequality $<, \leq, >, \geq$.

It is virtually impossible to try to "memorize" all the different cases.

You need to understand the distance concept to accurately get the right number line picture and reduce the problem!

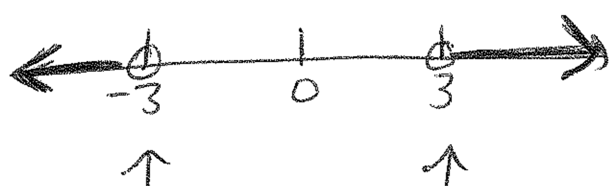
B. Some Examples With Positive Right Hand Side

Example 1: Solve $|3x-1|>3$ for x

Solution

Draw a picture!

Represent where distance from 0 is bigger than 3



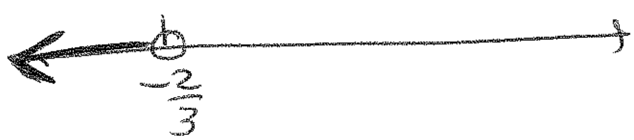
left point $3x-1<-3$ | right point $3x-1>3$

$$3x-1<-3 \quad \text{OR} \quad 3x-1>3$$

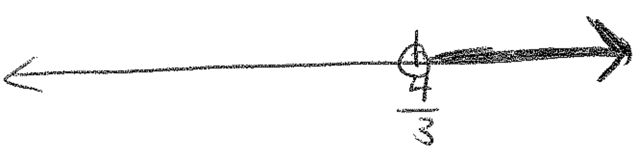
$$3x<-2 \quad \text{OR} \quad 3x>4$$

$$x<-\frac{2}{3} \quad \text{OR} \quad x>\frac{4}{3}$$

$x<-\frac{2}{3}$



$x>\frac{4}{3}$



OR

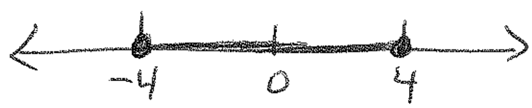
Ans: $\boxed{\left(-\infty,-\frac{2}{3}\right)\cup\left(\frac{4}{3},\infty\right)}$

Example 2: Solve $|2-x| \le 4$ for x

Solution

Draw a picture!

Represent where distance from 0 is less than or equal to 4



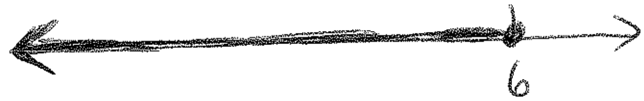
left point $2-x \ge -4$ right point $2-x \le 4$

$2-x \ge -4$ AND $2-x \le 4$

$-x \ge -6$ AND $-x \le 2$

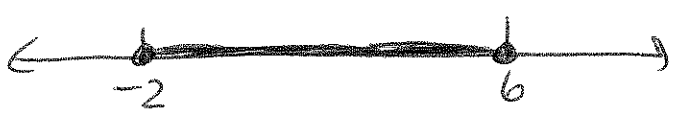
$x \le 6$ AND $x \ge -2$

$x \le 6$



$x \ge -2$

AND



Ans: $\boxed{[-2,6]}$

Example 3: Solve $4 < \left|x - \frac{11}{3}\right| + \frac{7}{3}$ for x

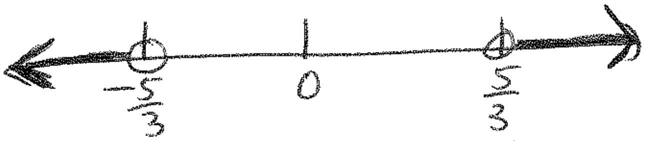
Solution

Rewrite: $\left|x - \frac{11}{3}\right| + \frac{7}{3} > 4$

Isolate: $\left|x - \frac{11}{3}\right| > \frac{5}{3}$ ⟵ $\left[4 - \frac{7}{3} = \frac{12}{3} - \frac{7}{3}\right]$

Draw a picture!

Represent where distance from 0 is bigger than $\frac{5}{3}$



left point — $x - \frac{11}{3} < -\frac{5}{3}$

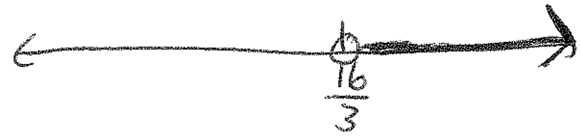
right point — $x - \frac{11}{3} > \frac{5}{3}$

$$x - \frac{11}{3} < -\frac{5}{3} \quad \underline{\text{OR}} \quad x - \frac{11}{3} > \frac{5}{3}$$

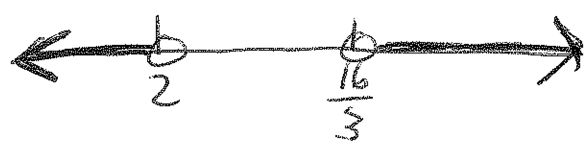
$$x < \frac{6}{3} \quad \text{OR} \quad x > \frac{16}{3}$$

$x < 2$

$x > \frac{16}{3}$



OR



Ans: $\boxed{(-\infty, -2) \cup \left(\frac{16}{3}, \infty\right)}$

Note: Clearing fractions at the start is hazardous. Absolute Value Bars are not parentheses.

C. Some Examples With Negative or Zero Right Hand Side

Example 1: Solve $|4+2x|<-2$ for x

Solution

Draw a picture!
Represent where distance is smaller than -2:

Impossible! Distance is never negative!

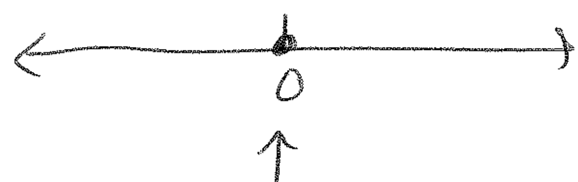
Ans: $\boxed{\emptyset}$

Example 2: Solve $|7-2x|\leq 0$ for x

Solution

Draw a picture!
Represent where distance is less than or equal to 0:

Distance can't be smaller than 0, and is 0 only at 0!



single point, no interval
thus one equation! [weird!!]

$$7-2x=0$$
$$-2x=-7$$
$$x=\frac{7}{2}$$

Ans: $\boxed{\left\{\frac{7}{2}\right\}}$

Example 3: Solve $|4+2x| \geq -3$ for x

Solution

Draw a picture!

Represent where distance is -3 or bigger:

Distance is always -3 or bigger (in particular, it's always 0 or bigger!)

Inequality always is true!

All real numbers!

Ans: $\boxed{(-\infty,\infty)}$

Example 4: Solve $|x-3|>0$ for x

Solution

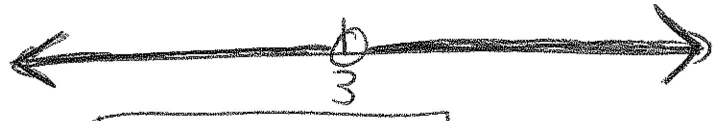
Draw a picture!

Represent where distance is positive!

This happens everywhere except at 0!

Thus all real numbers work except when $x-3=0$

i.e. all real numbers except $x=3$



Ans: $\boxed{(-\infty,3)\cup(3,\infty)}$

NOTE! A lot of other weird cases can happen. Picture is crucial!