

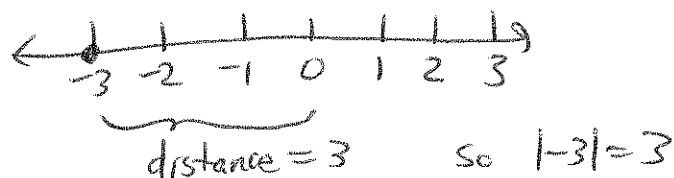
1.6F Review of Absolute Value and Absolute Value Equations

A. Absolute Value

1. Absolute Value of Numbers: "make it positive" $|-3| = 3$
2. True Definition (useful for solving equations and inequalities):

Distance from the origin

i.e. $|-3|$: locate -3 on number line



3. Piecewise Definition:

$$|x| = \begin{cases} x; & \text{if } x \geq 0 \\ -x; & \text{if } x < 0 \end{cases}$$

See MTH1825 Section 7.2C for problems and discussion for this. Necessary for Graded Hw#1

B. Absolute Value Equations

An equation with absolute values in it will usually split into two equations. To do this accurately, we need to use the true definition of absolute value in terms of distance from the origin

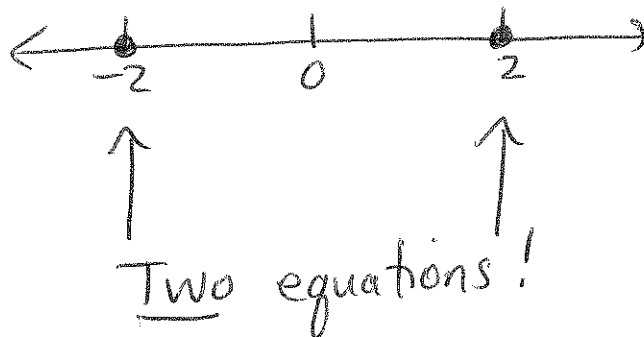
Strategy: Draw a number line picture that represents the distance.
Use the picture to rewrite without absolute values.

C. Examples with Positive Right Hand Side

Example 1: Solve $|3x+4|=2$ for x

Solution

Draw a picture showing where distance is equal to 2:



$$3x+4=-2 \quad \underline{\text{OR}} \quad 3x+4=2$$

$$3x=-6 \quad \underline{\text{OR}} \quad 3x=-2$$

$$x=-2 \quad \underline{\text{OR}} \quad x=-\frac{2}{3}$$

Ans: $\boxed{\{-2, -\frac{2}{3}\}}$

Example 2: Solve $2|6-x|-3=5$ for x

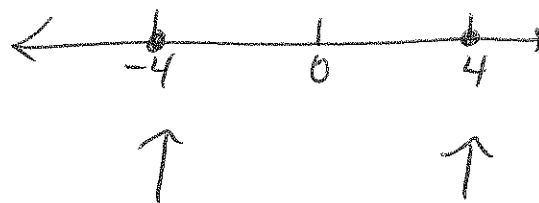
Solution

First we isolate the absolute value:

$$2|6-x|=8$$

$$|6-x|=4$$

Now we draw a picture showing where the distance is equal to 4:



Two equations!

$$6 - x = -4 \quad \text{OR} \quad 6 - x = 4$$

$$-x = -10 \quad \text{OR} \quad -x = -2$$

$$x = 10 \quad \text{OR} \quad x = 2$$

Ans: $\boxed{\{2, 10\}}$

D. Examples with Negative or zero Right Hand Side

Example 1: Solve $|3x + 4| = -2$ for x

Solution

Draw a picture showing where the distance is -2 ;

Impossible! Distance is never negative!

Ans: $\boxed{\emptyset}$

Example 2: Solve $|2x - 3| = 0$ for x

Solution

Draw a picture showing where the distance is 0:



one equation: $2x - 3 = 0 \Rightarrow x = \frac{3}{2}$

Ans: $\boxed{\{\frac{3}{2}\}}$

Example 3: Solve $2|6-x|+5=3$ for x

Solution

First we isolate the absolute value:

$$2|6-x| = -2$$

$$|6-x| = -1$$

Draw a picture showing where the distance is -1 :

Impossible! Distance is never negative!

Ans: $\boxed{\emptyset}$

NOTE

These examples show that it pays to think about distance, and not to just remember some $\pm$ rule.

E. Absolute Value on Two Sides

These will always split into two equations.

Use fact: Two distances are the same, if the numbers are the same, or they are negatives of each other.

Example: Solve $|1-2x|=|x+3|$ for x

Solution

Two equations: $1-2x=x+3$ OR $1-2x=-(x+3)$

$$-3x=2 \quad \text{OR} \quad 1-2x=-x-3$$

$$x=-\frac{2}{3} \quad \text{OR} \quad -x=-4$$

$$x=4$$

Ans: $\boxed{\{-\frac{2}{3}, 4\}}$