1.6 E Mixed Type Equations

A. Introduction

Some equations may be a mixture of several different types of equations. You need to combine methods!

B. Example

Solve $3x^{4/3} + 5x^{2/3} - 2 = 0$ for x

Solution

Clever substitution: $u = x^{2/3}$

Then $u^2 = x^{4/3}$, so we have

$$3u^2 + 5u - 2 = 0$$

Factor:

$$\begin{array}{c|l} 3u^2 + 5u - 2 = 0 & -6 \quad +, - \\ \hline 3u^2 + 6u - u - 2 = 0 & -6 \ \checkmark \\ 3u(u+2) - 1(u+2) = 0 & \\ (u+2)(3u-1) = 0 & \end{array}$$

$\Rightarrow u = -2$ OR $3u - 1 = 0$

$u = -2$ OR $u = \frac{1}{3}$

Get back to x's:

$$x^{2/3} = -2 \quad \text{OR} \quad x^{2/3} = \frac{1}{3}$$

Use Rational Index Principle:

$$X = \pm(-2)^{3/2} \quad \text{or} \quad X = \pm\left(\frac{1}{3}\right)^{3/2}$$

$$X = \pm(-1)^{3/2}2^{3/2} \quad \text{or} \quad X = \frac{\pm 1^{3/2}}{3^{3/2}}$$

$$X = \pm(\underbrace{\sqrt{-1}}_{i})^{3}2^{3/2} \quad \text{or} \quad X = \frac{\pm 1}{3^{3/2}}$$

$$X = \pm i^{3}2^{3/2} \quad \text{or} \quad X = \frac{\pm 1}{3^{3/2}}$$

$$X = \pm(-i)2^{3/2} \quad \text{or} \quad X = \frac{\pm 1}{3^{3/2}}$$

$$X = \mp i 2^{3/2} \quad \text{or} \quad X = \frac{\pm 1}{3^{3/2}}$$

Check:

$$X = \mp i 2^{3/2}$$

$$X^2 = (\mp i 2^{3/2})^2 = i^2 2^3$$

$$X^2 = -2^3 = -8$$

$$X^{2/3} = (-8)^{1/3} = -2$$

Thus

$$3X^{4/3} + 5X^{2/3} - 2$$

$$= 3(-2)^2 + 5(-2) - 2$$

$$= 3\cdot 4 - 10 - 2$$

$$= 0 \checkmark$$

$$X = \pm\frac{1}{3^{3/2}}$$

$$X^2 = \left(\frac{\pm 1}{3^{3/2}}\right)^2 = \frac{1}{3^3} = \frac{1}{27}$$

$$X^{2/3} = \left(\frac{1}{27}\right)^{1/3} = \frac{1}{27^{1/3}} = \frac{1}{3}$$

Thus

$$3X^{4/3} + 5X^{2/3} - 2$$

$$= 3\left(\frac{1}{3}\right)^2 + 5\left(\frac{1}{3}\right) - 2$$

$$= \frac{3}{9} + \frac{5}{3} - 2$$

$$= \frac{1}{3} + \frac{5}{3} - 2 = 0 \checkmark$$

$$\boxed{\text{Ans: } \left\{ \frac{-1}{3^{3/2}}, \frac{1}{3^{3/2}}, -i2^{3/2}, i2^{3/2} \right\}}$$