1.6A Introduction to Polynomial Equations

A. Introduction

Some (but not all) polynomial equations of degree higher than 2 can be solved by factoring. The strategy is the same as in 1.5A; namely get everything on one side, factor, and use the ZPP.

B. Examples

Example 1: Solve $x^3+x^2-20x=0$ for x

Solution

Factor: $x(x^2+x-20)=0$ can use AntiFOIL if you can't see it

$x(x+5)(x-4)=0$

ZPP: $x=0$ OR $x+5=0$ OR $x-4=0$

$x=0$ OR $x=-5$ OR $x=4$

Ans: $\boxed{\{-5,0,4\}}$

Example 2: Solve $x^3-3x^2-4x+12=0$ for x

Solution

Factor by grouping:

$x^2(x-3)-4(x-3)=0$

$(x-3)(x^2-4)=0$ difference of squares

$(x-3)(x+2)(x-2)=0$

Z.P.P (+ lazy): $x=3,-2,2$

Ans: $\boxed{\{-2,2,3\}}$

Example 3: Solve $y^7-y^3=8y^4-8$ for y

Solution

$$y^7-8y^4-y^3+8=0$$

Factor by grouping:

$$y^4(y^3-8)-1(y^3-8)=0$$

$$\underbrace{(y^3-8)}_{\text{diff. of cubes}}\underbrace{(y^4-1)}_{\text{diff. of squares}}=0$$

$$\left[(y-2)(y^2+2y+4)\right]\cdot\left[(y^2+1)\underbrace{(y^2-1)}_{\text{diff. of squares}}\right]=0$$

$$(y-2)(y^2+2y+4)(y^2+1)(y+1)(y-1)=0$$

Z.P.P.: $y=2$ OR $y^2+2y+4=0$ OR $y^2+1=0$ OR $y=-1$ OR $y=1$

Now $y^2+2y+4=0$:

$$\text{QF: } y=\frac{-2\pm\sqrt{4-4(1)(4)}}{2(1)}=\frac{-2\pm\sqrt{-12}}{2}$$

$$=\frac{-2\pm i\sqrt{12}}{2}=\frac{-2\pm 2i\sqrt{3}}{2}$$

$$=\frac{2(-1\pm i\sqrt{3})}{2}=-1\pm i\sqrt{3}$$

Now $y^2+1=0$: $y^2=-1 \Rightarrow y=\pm\sqrt{-1}=\pm i$

Ans: $\boxed{\{-1, 1, 2, -1-i\sqrt{3}, -1+i\sqrt{3}, -i, i\}}$

C. Comments

1. Observe in examples 1–3 that the total number of solutions matched the degree of the polynomial.

   This will always happen (if you count repeated solutions).

   This is called the <u>nth Root Theorem</u> and will be explored in more detail in Chapter 3.

2. Recall that factoring by grouping fails if the factor pairs don't match

   i.e. $3x^3 - x^2 - 8x - 4$

   $$x^2(3x-1) - 4(2x+1)$$

   not the same   Method Fails!

   In this case, try another arrangement.

   If all possible rearrangements fail, other techniques — for instance, later in Chapter 3 — must be used.