1.5H Discriminant II

A. Families of Quadratic Equations

Quadratic equations in x with one (or more) unspecified coefficients represent a whole family of quadratic equations

Example: $4x^2+kx+1=0$ represents the family of all quadratic equations with varying linear term

i.e. $4x^2+x+1=0$

$4x^2-2x+1=0$

$4x^2+\frac{3}{2}x+1=0$

$4x^2+1=0$

$4x^2-17x+1=0$ are all members of the family

B. Use of the Discriminant

1. The discriminant can be used to determine which members of the family have a certain "solution property".

2. In this case, you are not solving quadratic equations. Instead you are learning which quadratic equations are the desired ones, i.e. you are determining the coefficient

3. If an unspecified coefficient occurs in the x^2-term, then the value that wipes out the x^2-term must be handled separately (since if the x^2-term is gone, you don't have a quadratic equation!)

C. Examples

Example 1: For what values of k does the equation $4x^2+kx+1=0$ have one real (repeated) solution?

Solution

Use the discriminant!

$$D=b^2-4ac$$

$$D=k^2-4(4)(1)$$

$$D=k^2-16$$

Now one real (repeated) solution means $D=0$:

$$k^2-16=0$$

$$k^2=16$$

$$k=\pm 4$$

Ans: $\{-4,4\}$

Note: We are finding k, not solving the quadratic equation in x. Thus the answer means that $4x^2-4x+1=0$ and $4x^2+4x+1=0$ are the only quadratic equations in the family to have one real (repeated) solution. All others must have either two distinct real solutions or two distinct nonreal complex conjugate solutions.

Example 2: For what values of k does the equation $kx^2+3x+7=0$ yield 2 distinct real solutions?

Solution

Since k occurs in the x^2-term, we must handle $k=0$ separately

$k=0$: $\quad 3x+7=0 \quad \Longrightarrow$ one real solution (which we don't want)

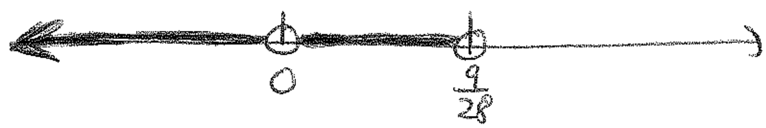
$k\neq 0$: Use the discriminant:

$$D=b^2-4ac$$
$$=3^2-4(k)(7)$$
$$=9-28k$$

Now 2 distinct real solutions means $D>0$:

$$9-28k>0$$
$$-28k>-9$$
$$k<\frac{9}{28}$$

Thus k must be less than $\frac{9}{28}$, BUT $k=0$ is bad (see above)



Ans: $\boxed{(-\infty,0)\cup\left(0,\frac{9}{28}\right)}$

Note: Again this means that any choice of k from this interval will give a quadratic equation with 2 distinct real solutions, i.e. $\frac{5}{28}x^2+3x+7=0$ is one

Example 3: For what values of k does the equation $(k-1)x^2+4kx+3=0$ have only one real solution? Assume that k is real.

Solution

Since $k=1$ wipes out the x^2-term, we treat this separately:

$k=1$: $4x+3=0 \Rightarrow$ one real solution ✓

$k\neq 1$: Use the discriminant:

$$\begin{aligned} D &= b^2-4ac \\ &= (4k)^2-4(k-1)(3) \\ &= 16k^2-12k+12 \end{aligned}$$

Now one real solution means $D=0$:

$$\begin{aligned} 16k^2-12k+12 &= 0 \\ 4(4k^2-3k+3) &= 0 \\ 4k^2-3k+3 &= 0 \qquad \text{[divide by 4]} \end{aligned}$$

By QF: $$k=\frac{3\pm\sqrt{9-4(4)(3)}}{2(4)}$$

$$=\frac{3\pm\sqrt{9-48}}{8}=\frac{3\pm\sqrt{-39}}{8}=\frac{3\pm i\sqrt{39}}{8}$$

However, these don't count because we said that k had to be real in this problem.

Thus the only value of k that worked was when $k=1$.

Ans: $\boxed{\{1\}}$