

1.5F Review of Quadratic Formula

A. Derivation of the Quadratic Formula

We can get a general formula for the solutions to $ax^2 + bx + c = 0$ by doing completing the square on the general equation.

$$ax^2 + bx + c = 0$$

$$a\left(x^2 + \frac{b}{a}x\right) + c = 0 \quad [\text{Factor out, first two}]$$

$$a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right] + c - a\left(\frac{b}{2a}\right)^2 = 0 \quad [\text{Completing the square}]$$

$$a\left(x + \frac{b}{2a}\right)^2 + c - a\left(\frac{b^2}{4a^2}\right) = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 = a\left(\frac{b^2}{4a^2}\right) - c$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - c$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - \frac{4ac}{4a}$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

B. Using the Quadratic Formula

Given $ax^2 + bx + c = 0$, we have $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 1: Solve $2x^2 - x + 3 = 0$ for x by the Quadratic Formula.

Solution

$$a = 2, \quad b = -1, \quad c = 3$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(3)}}{2(2)} \\ &= \frac{1 \pm \sqrt{1 - 24}}{4} \\ &= \frac{1 \pm \sqrt{-23}}{4} = \frac{1 \pm i\sqrt{23}}{4} \end{aligned}$$

Ans $\left\{ \frac{1 - i\sqrt{23}}{4}, \frac{1 + i\sqrt{23}}{4} \right\}$

Example 2: Solve $6x^2 - 2x - 1 = 0$ for x by the Quadratic Formula.

Solution

$$a = 6, \quad b = -2, \quad c = -1$$

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(6)(-1)}}{2(6)} \\&= \frac{2 \pm \sqrt{4 + 24}}{12} = \frac{2 \pm \sqrt{28}}{12}\end{aligned}$$

Now simplify:

$$x = \frac{2 \pm \sqrt{28}}{12} = \frac{2 \pm 2\sqrt{7}}{12} = \frac{2(1 \pm \sqrt{7})}{12} = \frac{1 \pm \sqrt{7}}{6}$$

Ans $\boxed{\frac{1 + \sqrt{7}}{6} \quad \text{or} \quad \frac{1 - \sqrt{7}}{6}}$

Example 3: Solve $\frac{5x}{x-2} + \frac{6}{(x-2)(x+3)} = \frac{x-1}{x+3}$ for x

Solution

Disallowed values: $x \neq 2, -3$

LCD = $(x-2)(x+3)$

Multiply both sides by LCD:

$$(x-2)(x+3) \left[\frac{5x}{x-2} + \frac{6}{(x-2)(x+3)} \right] = (x-2)(x+3) \left(\frac{x-1}{x+3} \right)$$

$$5x(x+3) + 6 = (x-2)(x-1)$$

$$5x^2 + 15x + 6 = x^2 - 3x + 2$$

Move everything to one side:

$$4x^2 + 18x + 4 = 0$$

$$2(2x^2 + 9x + 2) = 0$$

Divide by 2:

$$2x^2 + 9x + 2 = 0$$

Now use the quadratic formula:

$$a = 2, \quad b = 9, \quad c = 2$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-9 \pm \sqrt{(9)^2 - 4(2)(2)}}{2(2)} \\ &= \frac{-9 \pm \sqrt{81 - 16}}{4} = \frac{-9 \pm \sqrt{65}}{4} \end{aligned}$$

Neither of the solutions are disallowed.

Ans $\boxed{\left\{ \frac{-9 - \sqrt{65}}{4}, \frac{-9 + \sqrt{65}}{4} \right\}}$