1.5E Solving Quadratic Equations by Completing the Square

A. Introduction

Some quadratic equations can not be factored nicely, since the trinomial may be prime. If we use completing the square, we can solve all of them.

B. Method

1. Take the quadratic and complete the square.
2. Isolate the squared quantity (i.e. junk^2)
3. Solve using the square root principle.

C. Examples

Example 1: Solve $-3x^2 + 30x - 66 = 0$ for x

Solution

1. First complete the square:

$$-3(x^2 - 10x) - 66 = 0$$

$$-3(x^2 - 10x + 25) - 66 + 75 = 0$$

$$-3(x - 5)^2 + 9 = 0$$

2. Isolate the squared quantity:

$$-3(x-5)^2 = -9$$

$$(x-5)^2 = 3$$

3. Square Root Principle:

$$x - 5 = \pm\sqrt{3}$$

$$x = 5 \pm \sqrt{3}$$

Ans $\boxed{\{5-\sqrt{3},\, 5+\sqrt{3}\}}$

Example 2: Solve $9x^2 - 6x - 4 = 0$ for x

Solution

1. First complete the square:

$$9\left(x^2 - \frac{6}{9}x\right) - 4 = 0$$

$$9\left(x^2 - \frac{2}{3}x\right) - 4 = 0 \qquad \left[\left(\frac{-\frac{2}{3}}{2}\right)^2 = \left(-\tfrac{1}{3}\right)^2 = \tfrac{1}{9}\right]$$

$$9\left(x^2 - \frac{2}{3}x + \frac{1}{9}\right) - 4 - 1 = 0$$

$$9\left(x - \frac{1}{3}\right)^2 - 5 = 0$$

2. Isolate the squared quantity:

$$9\left(x - \frac{1}{3}\right)^2 = 5$$

$$\left(x - \frac{1}{3}\right)^2 = \frac{5}{9}$$

3. Square Root Principle:

$$x - \frac{1}{3} = \pm\sqrt{\frac{5}{9}} = \pm\frac{\sqrt{5}}{3}$$

$$x = \frac{1}{3} \pm \frac{\sqrt{5}}{3} = \frac{1 \pm \sqrt{5}}{3}$$

Ans $\boxed{\left\{\frac{1-\sqrt{5}}{3}, \frac{1+\sqrt{5}}{3}\right\}}$