1.5B Plus/Minus

A. $\pm$ and $\mp$

1. **Notation**

   $\pm$ plus or minus

   $\mp$ minus or plus

   These are a shorthand way of writing two solutions.

2. **Minus Signs**:

   $-(\pm) = \mp$

   $-(\mp) = \pm$

3. In expressions involving $\pm$ or $\mp$, we have two solutions.
   One by taking the "top" sign and one by taking the "bottom" sign.

B. Examples

Example 1: Write the individual expressions for $\dfrac{8 \mp \sqrt{2}}{4 \pm \sqrt{3}}$ and simplify.

Solution

a. "Top signs": $\dfrac{8 - \sqrt{2}}{4 + \sqrt{3}}$

Now rationalize:

$$\frac{8 - \sqrt{2}}{4 + \sqrt{3}} \cdot \frac{4 - \sqrt{3}}{4 - \sqrt{3}} = \frac{32 - 8\sqrt{3} - 4\sqrt{2} + \sqrt{6}}{16 - 3} = \boxed{\frac{32 - 8\sqrt{3} - 4\sqrt{2} + \sqrt{6}}{13}}$$

b. "Bottom signs": $\dfrac{8 + \sqrt{2}}{4 - \sqrt{3}}$

Now rationalize:

$$\frac{8 + \sqrt{2}}{4 - \sqrt{3}} \cdot \frac{4 + \sqrt{3}}{4 + \sqrt{3}} = \frac{32 + 8\sqrt{3} + 4\sqrt{2} + \sqrt{6}}{16 - 3} = \boxed{\frac{32 + 8\sqrt{3} + 4\sqrt{2} + \sqrt{6}}{13}}$$

Example 2: Simplify $6 - (4 \pm \sqrt{5})$ and simplify.

Solution

Use the distributive property and change all signs as required:

$$6 - (4 \pm \sqrt{5}) = 6 - 4 \mp \sqrt{5} = \boxed{2 \mp \sqrt{5}}$$

C. Comment on Difference

$\pm$ and $\mp$ are the same **provided** only **one** of them is present in an expression:

a. Thus $2 \mp \sqrt{5} = 2 \pm \sqrt{5}$.

b. However, $\dfrac{8 \mp \sqrt{2}}{4 \pm \sqrt{3}} \neq \dfrac{8 \pm \sqrt{2}}{4 \pm \sqrt{3}}$ (Why?)