1.5A Review of Quadratic Equations and Zero Product Principle

A. Definition of a Quadratic Equation

A quadratic equation is an equation of the form $ax^2+bx+c=0$
[like $3x^2-2x+4=0$]

Note: We require $a\neq 0$

B. Zero product Principle

If a product of factors is equal to zero, then one of the factors must be zero.

EX) If $(x-3)(2x+1)=0$, then $x-3=0$ OR $2x+1=0$
$\Rightarrow x=3$ OR $x=-\frac{1}{2}$

C. Solving Quadratic Equations By Factoring

Some (but not all) quadratic equations can be solved by factoring and using the Zero Product Principle

Method

1. Move everything to one side so that the other side is zero
2. Factor the trinomial
3. Set each factor to zero [Zero Product Principle]

D. Examples

Example 1: Solve $x^2+5x+6=0$ for x

Solution

Step 1 ✓

Step 2: $(x+2)(x+3)=0$ [Can use AntiFOIL here, if you can't see it]

Step 3: Zero Product Principle:

$$x+2=0 \quad \text{OR} \quad x+3=0$$

$$x=-2 \quad \text{OR} \quad x=-3$$

Ans: $\boxed{\{-3,-2\}}$

Example 2: Solve $2x^2-5x=12$ for x

Solution

Step 1: $2x^2-5x-12=0$

Step 2: Factor the trinomial [Review MTH1825 Sec 5.5]

$2x^2-5x-12=0$	-24 $+,-$
$2x^2+x-6x-12=0$	-6
$2x^2+2x-7x-12=0$	-14
$2x^2+3x-8x-12=0$	-24 ✓
$x(2x+3)-4(2x+3)=0$	
$(2x+3)(x-4)=0$	

Step 3: Zero Product Principle:

$$2x+3=0 \quad \text{OR} \quad x-4=0$$

$$x=\frac{-3}{2} \quad \text{OR} \quad x=4$$

Ans: $\boxed{\left\{\frac{-3}{2}, 4\right\}}$

Example 3: Solve $(x-1)(x-2)=2$ for x

Solution

Note: We can not use the Zero Product Principle now, since the right hand side is not zero!

Step 1: Expand: $x^2-3x+2=2$

Move to one side: $x^2-3x=0$

Step 2: Factor $x(x-3)=0$

Step 3: Zero Product Principle:

$x=0$ OR $x-3=0$

$x=0$ OR $x=3$

Ans: $\boxed{\{0,3\}}$

E. Comments

1. If the trinomial is not factorable, this method for solving quadratic equations fails.
2. We now review the square root principle and completing the square to handle the "unfactorable problem".