

1.2 Review of Rational Equations

A. Type

Equations with Rational Expressions (Algebraic Fractions) are called rational equations

e.g. $\frac{5}{x+1} + \frac{2}{x-3} = 6$

B. Method of Solution

1. Factor all denominators to determine the LCD
[Review MTH1825 Sections 5.4-5.7]
2. Determine the "disallowed values": These are the numbers that make the denominator zero [set denominators to zero to get these]
3. Multiply both sides by the LCD and solve the remaining equation
4. Throw out the answers in the disallowed value list.

Note: This is a completely different method from the method used to work non-equation problems such as

"Combine $\frac{5}{x+1} + \frac{2}{x-3}$ "

See MTH1825 Section 6.2 for these problems.

C. Examples

Example 1: Solve $\frac{3x}{x-1} + 2 = \frac{3}{x-1}$ for x

Solution

Step 1: ✓

Step 2: Set $x-1=0 \Rightarrow x=1$

Disallowed value: $x \neq 1$

Step 3: Multiply both sides by $LCD = x-1$:

$$(x-1) \left[\frac{3x}{x-1} + 2 \right] = (x-1) \left[\frac{3}{x-1} \right]$$

$$3x + 2(x-1) = 3$$

$$3x + 2x - 2 = 3$$

$$5x - 2 = 3$$

$$5x = 5$$

$$x = 1$$

Step 4: Our answer is disallowed!

Ans: $\boxed{\emptyset}$

Example 2: Solve $\frac{4}{3x-4} + \frac{2}{x+6} = \frac{5x}{3x^2+14x-24}$ for x

Solution

Step 1: Factor $3x^2+14x-24$:

Review MTH1825 Section 5.4

AntiFOIL:

$3x^2+14x-24$	$\begin{array}{c} \textcircled{-72} \\ +, - \end{array}$
$3x^2+15x-x-24$	-15
$3x^2+16x-2x-24$	-32
$3x^2+17x-3x-24$	-51
$3x^2+18x-4x-24$	$-72 \checkmark$

$3x(x+6)-4(x+6)$
 $(x+6)(3x-4)$

Trick: Frequently the trinomial is just the product of the other factors.

It's worth checking this first for quick factoring.

$$\frac{4}{3x-4} + \frac{2}{x+6} = \frac{5x}{(x+6)(3x-4)}$$

$$LCD = (x+6)(3x-4)$$

Step 2: Set $3x-4=0 \Rightarrow x=\frac{4}{3}$

Set $x+6=0 \Rightarrow x=-6$

Disallowed values: $x \neq \frac{4}{3}, -6$

Step 3: Multiply both sides by the LCD = $(x+6)(3x-4)$

$$(x+6)(3x-4) \left[\frac{4}{3x-4} + \frac{2}{x+6} \right] = (x+6)(3x-4) \left[\frac{5x}{(x+6)(3x-4)} \right]$$

$$4(x+6) + 2(3x-4) = 5x$$

$$4x + 24 + 6x - 8 = 5x$$

$$10x + 16 = 5x$$

$$5x = -16$$

$$x = \frac{-16}{5}$$

Step 4: not a disallowed value!

Ans: $\boxed{\left\{ \frac{-16}{5} \right\}}$

Example 3: Solve $\frac{4}{x-4} - \frac{3}{x+3} = \frac{x+24}{x^2-x-12}$ for x

Solution

Step 1: Factor $x^2 - x - 12$:

Using "Trick": $(x-4)(x+3)$

$$\frac{4}{x-4} - \frac{3}{x+3} = \frac{x+24}{(x-4)(x+3)}$$

$$\text{LCD} = (x-4)(x+3)$$

Step 2: Set $x-4=0 \Rightarrow x=4$

Set $x+3=0 \Rightarrow x=-3$

Disallowed values: $x \neq 4, -3$

Step 3: Multiply both sides by the LCD = $(x-4)(x+3)$:

$$(x-4)(x+3) \left[\frac{4}{x-4} - \frac{3}{x+3} \right] = (x-4)(x+3) \left[\frac{x+24}{(x-4)(x+3)} \right]$$

$$4(x+3) - 3(x-4) = x+24$$

$$4x+12-3x+12 = x+24$$

$$x+24 = x+24$$

$$24 = 24$$

All real numbers!

Step 4: All real numbers except $x=4$ and $x=-3$



Use interval notation!

Ans: $\boxed{(-\infty, -3) \cup (-3, 4) \cup (4, \infty)}$