1.2 B Interval Notation and Linear Inequalities Review

A. Interval Notation

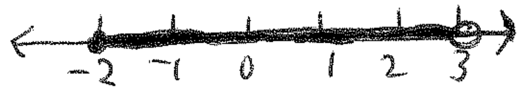
$(-\infty, \infty)$ is an interval

If a problem has a whole range of solutions, we write the solution in <u>interval notation</u>.

$[$ = include the point "closed"

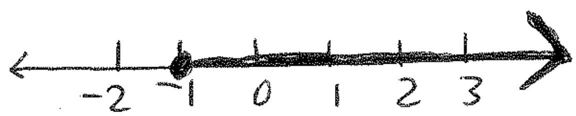
$($ = don't include the point "open"

<u>Example 1</u>: Write the shaded region in interval notation



<u>Ans</u>: $[-2, 3)$

<u>Example 2</u>: Write the shaded region in interval notation



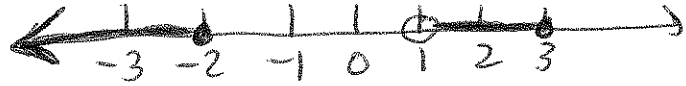
<u>Ans</u>: $[-1, \infty)$

<u>Note</u>: ① You always write intervals with the smallest number first

② You never include ∞ (or $-\infty$)
i.e. these are never "bracketed" [ ]

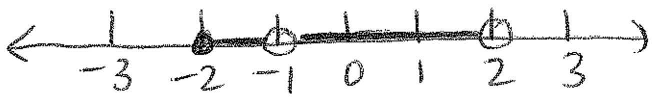
If a shaded region is broken into pieces, you write an interval for each piece. Then join them with a union symbol "$\cup$".

Example 3: Write the following shaded regions in interval notation



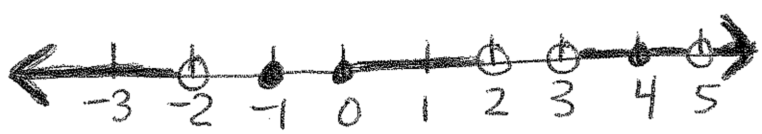
Ans: $\boxed{(-\infty,-2] \cup (1,3]}$

Example 4: Write the following shaded regions in interval notation



Ans: $\boxed{[-2,-1) \cup (-1,2)}$

Example 5: Write the following shaded regions in interval notation

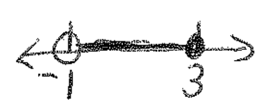
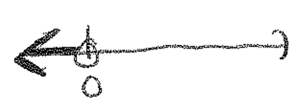


Ans: $(-\infty,-2) \cup [-1,-1] \cup [0,2) \cup (3,5) \cup (5,\infty)$

OR

$(-\infty,-2) \cup \{-1\} \cup [0,2) \cup (3,5) \cup (5,\infty)$

B. Set-Builder Notation

Interval	Set-Builder	Graph
$(-1,3]$	$\{x \mid -1 < x \leq 3\}$	
$[2,\infty)$	$\{x \mid x \geq 2\}$	
$(-\infty,0)$	$\{x \mid x < 0\}$	

C. Linear Inequalities

Same as linear equations, except instead of $=$, we have

$$<, \le, >, \ge$$

To solve: We use the same techniques as for linear equations except that the inequality sign switches direction if we multiply or divide by a negative number.

The solution, in general, will be an interval.

Example: Solve $3(5-x) \le 2x+4$ for x

Solution

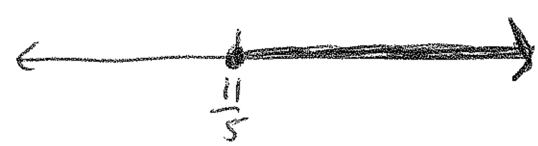
$$15 - 3x \le 2x + 4$$

$$-5x \le -11$$

$$x \ge \frac{-11}{-5} \qquad \text{divide by negative: switch!}$$

$$x \ge \frac{11}{5}$$

Graph:



Ans: $\boxed{\left[\frac{11}{5}, \infty\right)}$

D. Triple Inequalities

Three pieces with two inequality signs:

1. Smaller number on left
2. "Jaws" face right

e.g. $-2 \le 2x+3 < 7$

$0 < 3 - 5x \le 8$

E. Solving Triple Inequalities

Any manipulation is done to all components.
If we multiply/divide by a negative number, all inequalities switch.

F. Examples

Example 1: Solve $-2 \le 5-2x < 1$ for x

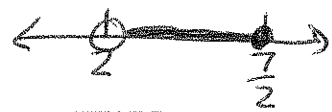
Solution

$-7 \le -2x < -4$ [subtract 5]

$\frac{-7}{-2} \ge x > \frac{-4}{-2}$ [divide by negative]

$\frac{7}{2} \ge x > 2$

Rewrite: $2 < x \le \frac{7}{2}$



Ans: $(-2, \frac{7}{2}]$

Example 2: Solve $4 < 4 - \frac{1}{3}x \le 8$ for x

Solution

Clear fractions by multiplying all by 3:

$12 < 12 - x \le 24$

$0 < -x \le 12$ [subtract 12]

$0 > x \ge -12$ [divide by -1]

Rewrite: $-12 \le x < 0$

Ans: $\boxed{[-12, 0)}$